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**LATERAL VIBRATION  
CHARACTERISTICS OF THE 1/10-SCALE  
APOLLO/SATURN V REPLICA MODEL**

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SUMMARY

A 1/10-scale replica model of the Apollo/Saturn V vehicle has been tested to determine its free-free lateral vibration characteristics. Several simulated propellant loadings were investigated for each of three model configurations simulating the first-stage, second-stage, and third-stage burns. Tests were conducted in both the pitch and yaw directions. A one-dimensional lumped-parameter analysis employing the transfer matrix method was used to calculate the uncoupled natural modes and frequencies of the system. Analytical parameter studies were conducted to investigate the effects of engine flexibility, liquid slosh, and decreased stiffness due to ineffective skin on the modal behavior of the system.

For modes in which the model responds as a beam, the analysis predicts the behavior with reasonable accuracy. The presence of large concentrated masses such as contained liquids or concentrated structural and subsystem masses tends to limit the number of modes which the analysis can be expected to produce. Branch masses which possess resonant frequencies near structural resonant frequencies can impair experimental definition of the structural resonant frequency. Likewise, the existence of shell modes in the primary structure makes experimental definition of higher mode shapes and frequencies difficult. The one-dimensional analysis cannot predict radial or shell response; therefore, for applications requiring knowledge of the shell behavior, analyses which include two- or three-dimensional motions are required.

INTRODUCTION

Analytical representations of the dynamics of complex launch-vehicle structures involve many idealizations and simplifying assumptions. Thus, at some point in the vehicle development, analytical methods and procedures proposed for determining the launch-vehicle dynamic behavior must be verified. Verification is usually obtained by comparing the experimental data from full-scale ground tests of the vehicle, and ultimately flight data, with the predictions of the analysis.

Dynamic models have been used in investigations of aeroelastic stability phenomena (see, for example, refs. 1 to 3) and it is natural to extend the concept to replica models of launch vehicles. Utilizing a 1/5-scale model, the concept was applied to the Saturn I launch vehicle as reported in references 4 to 9. This work verified the premise that the overall dynamic characteristics of large, complex vehicles can be accurately, economically, and quickly determined through model tests. Confidence in the use of dynamic models was further advanced by data obtained during scaled model studies of an operational Air Force launch vehicle, as reported in references 10 and 11.

Confidence in launch-vehicle analyses is enhanced by good correlation of these analyses with model experimental data. Also, areas of applicability and limitations of analytical models can be determined effectively early in the launch-vehicle development cycle through such correlations. One method of analyzing liquid-propellant launch vehicles is to represent the launch vehicle as a beam with attached spring-mass components. Simplicity and ease of application make this representation attractive in studies in which overall dynamic characteristics are desired – for example, in control-system stability investigations. When such a representation is verified by correlation with experimental data, parametric studies to assess the effects of shear deformation, rotatory inertia, and slosh can be performed efficiently. This analytical approach, however, has one important limitation in that it has no provision for the inclusion of shell-type modes.

Experimental and analytical studies of the dynamic response characteristics of the Apollo/Saturn V launch vehicle have been conducted with scale models. The models employed were: A 1/40-scale dynamic model of the launch vehicle, launch platform, and umbilical tower; and a 1/10-scale structural replica model of the launch vehicle.

The purposes of the model construction, test, and analysis program were: (1) To advance the state of the art in model construction; (2) to provide structural dynamics data to organizations responsible for full-scale analysis, design, and test prior to availability of full-scale test data; (3) to provide information on the practical limits of scaling; and (4) to provide experimental vibration data for evaluation of vibration analyses. Results from the 1/40-scale dynamic-model program are reported in references 12 to 14. The longitudinal dynamic characteristics of the 1/10-scale replica model are given in reference 15.

The purpose of this paper is to present the results of a combined experimental and analytical study of the pitch and yaw vibration characteristics of the 1/10-scale replica model of the Apollo/Saturn V launch vehicle for simulated flight times ranging from lift-off through third-stage burnout. These results help to establish the areas of applicability of the one-dimensional lumped-parameter representation of a typical launch-vehicle structure.

## SYMBOLS

|                   |   |
|-------------------|---|
| $A$               | area  |
| $a_{\text{ref}}$  | acceleration at reference station, divided by gravitational acceleration  |
| $C$               | integration constant  |
| $c_1, c_2, \dots$ | flexibility influence coefficients  |
| $d$               | suspension-cable separation distance (see fig. 4)                         |
| $E$               | Young's modulus   |
| $[e]_i$           | transfer matrix for $i$ th elastic beam                                   |
| $f$               | frequency   |
| $G$               | shear modulus   |
| $g$               | structural damping coefficient  |
| $h$               | amplitude of total bending and shear deformation                          |
| $I$               | area moment of inertia of cylindrical shell                               |
| $\{I\}$           | column matrix each element of which is unity                              |
| $I_{0,i}$         | second moment of $i$ th concentrated mass about its own center of gravity |
| $i = \sqrt{-1}$   |   |
| $K$               | form factor for shear deformation   |
| $l_i$             | length of $i$ th elastic beam element                                     |
| $M$               | bending moment  |
| $M_{gh}$          | normalized generalized mass   |

|                      |  |
|----------------------|--|
| $M_t$                | total mass of configuration                                |
| $\bar{M}_{gg}$       | nondimensional generalized mass                            |
| $m(x)$               | mass distribution  |
| $m_{gh}$             | generalized mass corresponding to gth and hth modes        |
| $m_i$                | ith concentrated mass                                      |
| $[m(\omega)]_i$      | transfer matrix for ith concentrated mass                  |
| $R$                  | radius of cylindrical shell                                |
| $r_i$                | offset of the ith concentrated mass                        |
| $t$                  | time   |
| $[U(\lambda)]$       | transfer matrix  |
| $[\bar{U}(\lambda)]$ | reduced-order transfer matrix                              |
| $V$                  | transverse shear   |
| $X$                  | deflection function  |
| $x$                  | Cartesian coordinate                                       |
| $x/L$                | dimensionless longitudinal coordinate of configuration I   |
| $x'/L'$              | dimensionless longitudinal coordinate of configuration II  |
| $x''/L''$            | dimensionless longitudinal coordinate of configuration III |
| $y$                  | lateral displacement                                       |
| $\{Z\}$              | state vector   |
| $\{\bar{Z}\}$        | reduced-order state vector                                 |

|                          |   |
|--------------------------|---|
| $\alpha$                 | incremental bending deformation                                       |
| $\beta$                  | incremental shear deformation   |
| $\delta$                 | tangential deviation  |
| $\epsilon$               | neutral-axis offset   |
| $\theta$                 | neutral-axis offset angle, $\sin^{-1}\left(\frac{\epsilon}{R}\right)$ |
| $\lambda$                | frequency parameter   |
| $\xi$                    | dummy variable  |
| $\rho$                   | mass density  |
| $[\sigma(\omega)]$       | collapsed transfer matrix   |
| $[\bar{\sigma}(\omega)]$ | reduced-order transfer matrix   |
| $\tau$                   | skin thickness  |
| $\phi$                   | slope of the bending deformation                                      |
| $\omega$                 | circular frequency  |

**Subscripts:**

|   |                   |
|---|-------------------|
| a | analytical        |
| B | branched beam     |
| g | gth mode          |
| h | hth mode          |
| i | ith mass location |
| j | jth mass location |

|     |  |
|-----|--|
| $l$ | last mass in chain of transfer matrices  |
| $0$ | first mass in chain of transfer matrices |
| $p$ | pitch                                    |
| $R$ | root end of branched beam                |
| $T$ | free end of branched beam                |
| $y$ | yaw                                      |

Matrix notation:

|  |                              |
|--|------------------------------|
| $\begin{bmatrix} \end{bmatrix}$            | square or rectangular matrix |
| $\{ \}$                                    | column matrix                |
| $\text{adj} \begin{bmatrix} \end{bmatrix}$ | adjoint matrix               |

A superscript  $T$  denotes a transpose matrix.

Dots over symbols denote differentiation with respect to time.

## APPARATUS AND TEST PROCEDURE

### Model Description

A complete description of the 1/10-scale Apollo/Saturn V model, the design philosophy, typical problems encountered in the fabrication along with their solutions, and a discussion of associated test equipment are presented in reference 16. The model description presented in this paper will, therefore, be brief and will illustrate only the general model configuration. All nomenclature used in discussing the model components will be the same as that for the full-scale counterparts.

The complete model, shown photographically in figure 1 and schematically in figure 2, consists of the S-IC, S-II, and S-IVB stages, the instrument unit (IU), and the Apollo payload with its launch escape system (LES). All main, load-carrying structures of the model are essentially geometrically scaled. However, the model payload differs significantly from full-scale hardware. For example, the model Apollo command module (CM) and service module (SM) retain only general external dimensions, longitudinal mass

distribution, and bending stiffnesses as scaled quantities. The simulated lunar module (LM) has only the correct mass, center of mass, mass moments of inertia, and simulated truss work for attachment of the LM to the adapter structure. The payload dimensions, inertias, and stiffness and mass distributions were scaled from the CM, SM, LM, and Saturn LM adapter (SLA) concepts existing at the time the model was constructed and are, therefore, not representative of any flight configuration.

The calculated cross-sectional area distribution of the models is given in figure 3 and measured mass characteristics of the three configurations tested are given in table 1. A discussion of the method used in calculating the model stiffness from the cross-sectional area plot is given in a subsequent section. The three configurations are defined numerically in the order in which they occur in flight. Configuration I consists of the complete model, configuration II is the model with the S-IC stage removed, and configuration III is the model with both the S-IC and S-II stages removed. In flight, the LES is not jettisoned until the S-II engines reach full thrust. Therefore an additional subconfiguration, representing the vehicle immediately following the S-IC staging, was studied.

#### Liquid Propellant Simulation

Onboard liquids in the three booster stages were simulated as follows: liquid oxygen (lox) in all stages and the RP-1 fuel in the first stage were simulated with water; the liquid hydrogen (LH<sub>2</sub>) in the second and third stages was simulated with small, hollow styrene plastic beads. Proper control of the water and bead levels then permitted representation of the correct propellant weight; however, some deviation from full-scale mass distribution occurred as a result of differences in specific gravity between water (1.0) and RP-1 (0.80) and lox (1.14).

#### Suspension System

Two-cable suspension systems of the type described in reference 17 and depicted schematically in figure 4 were used to support the models and to provide simulated free-free restraint. The configuration under test was mounted on a cradle at its base and two vertical cables were connected to the cradle so that the plane of the cables passed through the model center line normal to the direction of excitation. Tipover stability was provided by horizontal restraint cables connecting the support cables to the model at a point above the center of gravity of the configuration. The tension in the horizontal cables, and thus the stabilizing force, was controlled by adjustment of the cable terminal separation distance, designated  $d$  in the sketches. The cables were steel, composed of seven 19-wire strands, and were 5/16 inch in diameter. All rigid-body frequencies ( $f < 1.0$  Hz) were much lower than the lowest structural frequency ( $f = 8.6$  Hz) for all tests. Force application points for the various tests are also shown in figure 4.

TABLE I.- MEASURED MASS DATA FOR 1/10-SCALE SATURN V MODEL

| Model section   | c.g., $\frac{x}{L}$ | Struct. mass, kg | Configuration I      |          |                     |          |                     |          | Struct. mass, kg | Configuration II     |          |                     |          |                     |          | Struct. mass, kg | Configuration III    |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
|-----------------|---------------------|------------------|----------------------|----------|---------------------|----------|---------------------|----------|------------------|----------------------|----------|---------------------|----------|---------------------|----------|------------------|----------------------|----------|---------------------|----------|---------------------|----------|--|--|--|--|--|--|--|--|
|                 |                     |                  | Fuel and lox loading |          |                     |          |                     |          |                  | Fuel and lox loading |          |                     |          |                     |          |                  | Fuel and lox loading |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
|                 |                     |                  | 100 percent          |          | 50 percent          |          | 0 percent           |          |                  | 100 percent          |          | 50 percent          |          | 0 percent           |          |                  | 100 percent          |          | 50 percent          |          | 0 percent           |          |  |  |  |  |  |  |  |  |
|                 |                     |                  | c.g., $\frac{x}{L}$  | Mass, kg | c.g., $\frac{x}{L}$ | Mass, kg | c.g., $\frac{x}{L}$ | Mass, kg |                  | c.g., $\frac{x}{L}$  | Mass, kg | c.g., $\frac{x}{L}$ | Mass, kg | c.g., $\frac{x}{L}$ | Mass, kg |                  | c.g., $\frac{x}{L}$  | Mass, kg | c.g., $\frac{x}{L}$ | Mass, kg | c.g., $\frac{x}{L}$ | Mass, kg |  |  |  |  |  |  |  |  |
| Payload:        |                     |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| LES             | 0.946               | 3.96             |                      |          |                     |          |                     |          | 3.96             |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Command module  | .894                | 3.02             |                      |          |                     |          |                     |          | 3.02             |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Service module  | .861                | 22.39            |                      |          |                     |          |                     |          | 22.39            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| LM and adapter  | .788                | 14.96            |                      |          |                     |          |                     |          | 14.96            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Instrument unit | .761                | 1.66             |                      |          |                     |          |                     |          | 1.66             |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-IVB:          |                     |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-IVB hydrogen  |                     |                  | 0.705                | 18.4     | 0.705               | 18.4     | 0.705               | 18.4     |                  | 0.705                | 18.4     | 0.705               | 18.4     | 0.705               | 18.4     |                  | 0.705                | 18.4     | 0.686               | 9.19     | 0                   | 0        |  |  |  |  |  |  |  |  |
| S-IVB tanks     | .692                | 7.32             |                      |          |                     |          |                     |          | 7.32             |                      |          |                     |          |                     |          |                  | 7.32                 |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-IVB lox       |                     |                  | .655                 | 88.0     | .655                | 88.0     | .655                | 88.0     |                  | .655                 | 88.0     | .655                | 88.0     | .655                | 88.0     |                  | .655                 | 88.0     | .648                | 44.0     | 0                   | 0        |  |  |  |  |  |  |  |  |
| Aft structure   | .622                | 4.75             |                      |          |                     |          |                     |          | 4.75             |                      |          |                     |          |                     |          |                  | 4.75                 |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| J-2 engine      | .615                | 1.42             |                      |          |                     |          |                     |          | 1.42             |                      |          |                     |          |                     |          |                  | 1.42                 |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Support cradle  | .588                |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  | 4.3                  |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-II:           |                     |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-II hydrogen   |                     |                  | .514                 | 70.8     | .514                | 70.8     | .514                | 70.8     |                  | .514                 | 70.8     | .486                | 35.4     | 0                   | 0        |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-II tanks      | .482                | 23.61            |                      |          |                     |          |                     |          | 23.61            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-II lox        |                     |                  | .431                 | 344.7    | .431                | 344.7    | .431                | 344.7    |                  | .431                 | 344.7    | .421                | 172.4    | 0                   | 0        |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Aft structure   | .392                | 13.03            |                      |          |                     |          |                     |          | 13.03            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| J-2 engines     | .384                | 23.61            |                      |          |                     |          |                     |          | 23.61            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Support cradle  | .358                |                  |                      |          |                     |          |                     |          | 4.3              |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| S-IC:           |                     |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Lox tank        | .259                | 20.35            | .262                 | 1343     | .227                | 671.3    | 0                   | 0        |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Inter tank      | .181                | 14.32            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Fuel tank       | .120                | 10.04            | .109                 | 607.8    | .086                | 303.9    | 0                   | 0        |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Aft structure   | .045                | 47.31            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| F-1 engines     | .011                | 40.28            |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |
| Support cradle  | .021                | 6.92             |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |                  |                      |          |                     |          |                     |          |  |  |  |  |  |  |  |  |

| Configuration mass totals |                             |          |
|---------------------------|-----------------------------|----------|
| Configuration             | Propellant loading, percent | Mass, kg |
| I                         | 100                         | 2371.7   |
|                           | 50                          | 1756.1   |
|                           | 0                           | 780.9    |
| II                        | 100 + LES                   | 649.9    |
|                           | 100                         | 645.9    |
|                           | 50                          | 438.2    |
|                           | 0                           | 230.4    |
| III                       | 100                         | 170.2    |
|                           | 50                          | 117.0    |
|                           | 0                           | 63.8     |

## Instrumentation

The instrumentation of the 1/10-scale Saturn V model was designed to provide continuous electronic signals to define the dynamic response of the model to sinusoidal force inputs. The instrumentation system provided a means of monitoring exciter inputs and transducer output signals, recording these signals for subsequent in-depth analysis, and performing simple onsite data analysis. The locations of primary transducers for sensing responses of the models to the induced excitation are shown in figure 5. The primary transducers used in the test program were Kistler Model 303 servo accelerometers. Unless otherwise noted, these accelerometers were mounted tangentially on the model with the sensitive axis parallel to the direction of the input force vector. Responses normal to the plane of excitation were also measured at selected locations. A typical accelerometer installation is shown photographically in figure 6. Whenever supplementary data were required, additional measurements were made with a vacuum-mounted movable accelerometer. Accelerometers were also mounted on all engines and on the simulated LM.

## Data Acquisition System

The layout of the data acquisition system is shown in figure 7. Data signal cables from the transducers were connected to data group switches in prearranged combinations so that a group of 12 selected channels of information could be monitored or recorded simultaneously. Three data channels were common to all groups: the force-gage output, exciter-oscillator output, and tip-accelerometer signal. Any channel within a given group could be monitored on oscilloscopes, rms meter, or x-y plotter. The data were also recorded on analog tape for subsequent data reduction. The group of instruments designated "Onsite analysis" permitted preliminary, quick-look evaluation of either real-time or tape-recorded data signals.

## Data-Reduction Procedure

Recorded data were converted by means of a direct 24-point-per-cycle conversion from analog to digital format suitable for computer-mechanized reduction. Data samples were subjected to quality-control analysis and then used to construct a single cycle of data considered to be representative of the set from which it originated. This constructed cycle was then harmonically analyzed to determine the magnitudes of the fundamental through tenth harmonics and their respective phase angles with reference to that of the input force. The normalized amplitude of the fundamental harmonic, when plotted as a function of the transducer coordinate, yields the normalized deflection at that location.

To obtain damping constants, power to the shaker armature was switched off and the amplitude decay of one or more selected transducer signals was recorded as an

oscillogram. The amplitudes of at least 10 cycles of the resulting decay signal were used to determine the damping value.

### Test Procedure

The experimental data were obtained in basically the same manner for all configurations and test conditions. The input-force generator supplied a constant-amplitude sinusoidal force to the model at slowly increasing frequency. Selected transducer outputs were automatically plotted as diagrams of total amplitude versus frequency. These signals were also simultaneously displayed as Lissajous patterns on an oscilloscope, and peak response frequencies and phase shifts were noted. Examination of several diagrams of amplitude versus frequency served to identify individual peak response frequencies, which were then examined by manually tuning the frequency of excitation to obtain peak amplitude and recording the output of all transducers on analog tape for subsequent harmonic analysis. Damping data were then taken as previously discussed.

### ANALYSIS

A theoretical investigation was made, in conjunction with the experimental program, to gain insight into the degree of analytical refinement required to produce reasonably accurate calculated mode shapes and natural frequencies. Characteristics calculated for the analytical model, and therefore the correlation achieved between measured and calculated results, depend on both the nature of the mathematical model and the method used to analyze it. Therefore, the subsequent study can be interpreted as a measure of the adequacy of the mathematical model and of the analytical procedure as well. The matrix-Holzer method, which is employed in the calculations, has been used in various forms for many years. Verification of its suitability for analyzing various beam-type structures has been reported (refs. 18 and 19). Nevertheless, few explicit comparisons between measured and calculated characteristics for structures typical of launch vehicles are present in the open literature.

### General Method

The development of the transfer matrix method for flexural vibration of beams is presented in chapter 5 of reference 20. The following treatment is similar except that it is stated here in a manner intended to show greater parallel with the classical solution for a beam with distributed mass and distributed stiffness. The development illustrates also that the transfer matrix method is not restricted to discrete systems.

A variable-coefficient partial differential equation is considered which is dependent on a single space variable and a time variable. In addition, the solution must satisfy

given boundary conditions at two separate points. The equation considered is that which describes the lateral displacement of a beam with shear deflection and rotatory inertia effects included:

$$\frac{\partial^4 y}{\partial x^4} + \frac{m(x)}{E(x)I(x)} \frac{\partial^2 y}{\partial t^2} - \left[ \frac{\rho(x)}{E(x)} + \frac{\rho(x)}{K(x)G(x)} \right] \frac{\partial^4 y}{\partial x^2 \partial t^2} = 0 \quad (1)$$

Solutions of this equation may be assumed as a product of a space-dependent part and a time-dependent part in the form

$$y(x,t) = X(x)e^{i\omega t} \quad (2)$$

Equation (1) is thus reduced to an ordinary differential equation in the space variable with the frequency  $\omega$  as a parameter:

$$\frac{d^4 X(x)}{dx^4} - \frac{m(x)}{E(x)I(x)} \omega^2 X(x) + \left[ \frac{\rho(x)}{E(x)} + \frac{\rho(x)}{K(x)G(x)} \right] \omega^2 \frac{d^2 X(x)}{dx^2} = 0 \quad (3)$$

If the coefficients in equation (3) are independent of  $x$ , the general solution for  $X(x)$  will be of the form

$$X(x) = Ce^{\lambda x} \quad (4)$$

Since the differential equation is of fourth order, the substitution of equation (4) into equation (3) will yield a quartic equation in  $\lambda$ . There will then be four roots of the quartic

$$\lambda_{1,2,3,4} = \pm \sqrt{\frac{m\omega^2}{2EI}} \pm \frac{1}{2} \sqrt{\left( \frac{m\omega^2}{EI} \right)^2 + \frac{4\rho\omega^2}{E} \left( 1 + \frac{E}{KG} \right)} \quad (5)$$

and four undetermined coefficients  $C$  so that the general solution may be written

$$X(x) = \sum_{s=1}^4 C_s e^{\lambda_s x} \quad (6)$$

A set of four equations necessary for determining the four constants of integration is formed from derivatives of  $X(x)$ :

$$X(x) = \sum_{s=1}^4 C_s e^{\lambda_s x} \quad (7a)$$

$$\frac{dX(x)}{dx} = \sum_{s=1}^4 \lambda_s C_s e^{\lambda_s x} \quad (7b)$$

$$\frac{d^2 X(x)}{dx^2} = \sum_{s=1}^4 \lambda_s^2 C_s e^{\lambda_s x} \quad (7c)$$

$$\frac{d^3 X(x)}{dx^3} = \sum_{s=1}^4 \lambda_s^3 C_s e^{\lambda_s x} \quad (7d)$$

These equations may be written in matrix notation as

$$\left\{ \frac{d^{p-1} X(x)}{dx^{p-1}} \right\} = \left[ \lambda_s^{p-1} e^{\lambda_s x} \right] \{C_s\} \quad (p, s = 1, 2, 3, 4) \quad (8)$$

where, by definition,

$$\frac{d^0 X(x)}{dx^0} = X(x)$$

The constants of integration will be determined from the boundary conditions prescribed in this case at the two end points of the beam,  $x = 0$  and  $x = l$ .

A relationship can be found between the dependent variable  $X(x)$  and its derivatives at stations  $x = 0$  and  $x = l$ :

$$\left\{ \frac{d^{p-1} X(x)}{dx^{p-1}} \right\}_{x=0} = \left[ \lambda_s^{p-1} \right] \{C_s\} \quad (9)$$

and

$$\left\{ \frac{d^{p-1} X(x)}{dx^{p-1}} \right\}_{x=l} = \left[ \lambda_s^{p-1} e^{\lambda_s l} \right] \{C_s\} \quad (10)$$

Upon eliminating the integration constants, the result is

$$\left\{ \frac{d^{p-1} X(x)}{dx^{p-1}} \right\}_{x=l} = \left[ \lambda_s^{p-1} e^{\lambda_s l} \right] \left[ \lambda_s^{p-1} \right]^{-1} \left\{ \frac{d^{p-1} X(x)}{dx^{p-1}} \right\}_{x=0} \quad (11)$$

We may express this equation in the notation used in reference 20 as follows:

$$\{Z\}_l = [U(\lambda_s)] \{Z\}_0$$

where  $\{Z\} = \{X, \phi, V, M\}^T$ . The quantities  $V$  and  $M$  are proportional to  $d^3 X/dx^3$  and  $d^2 X/dx^2$ , respectively. The quantity  $\{Z\}$  is referred to as the state vector because it describes the state of the beam at a given cross section and  $[U(\lambda_s)]$  is the transfer matrix given by

$$[U(\lambda_s)] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_1^{2EI} & \lambda_2^{2EI} & \lambda_3^{2EI} & \lambda_4^{2EI} \\ \lambda_1^{3EI} & \lambda_2^{3EI} & \lambda_3^{3EI} & \lambda_4^{3EI} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 l} & & & \\ & e^{\lambda_2 l} & & \\ & & e^{\lambda_3 l} & \\ & & & e^{\lambda_4 l} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_1^{2EI} & \lambda_2^{2EI} & \lambda_3^{2EI} & \lambda_4^{2EI} \\ \lambda_1^{3EI} & \lambda_2^{3EI} & \lambda_3^{3EI} & \lambda_4^{3EI} \end{bmatrix}^{-1} \quad (12)$$

The solution is found by imposing the boundary conditions at  $x = 0$  and  $x = l$ . This leads to a homogeneous equation

$$\{0\}_l = [\bar{U}(\lambda_s)] \{\bar{Z}\}_0 \quad (13)$$

The matrix  $[\bar{U}(\lambda_s)]$  and the vector  $\{\bar{Z}\}_0$  are one-half the order of  $[U(\lambda_s)]$  and  $\{Z\}_0$ . A nontrivial solution for  $\{\bar{Z}\}_0$  from equation (13) exists if and only if the determinant  $|\bar{U}(\lambda_s)| = 0$ . The  $\lambda_s$  are functions of  $\omega$  as given in equation (5); consequently there will be a vector  $\{\bar{Z}\}_0$  for each value of  $\omega$  for which  $|\bar{U}(\lambda_s)| = 0$ . The space-dependent function is then formed by combining equations (6) and (9) and using the state vector  $\{\bar{Z}\}_0$  related to each natural frequency  $\omega$ . The result is

$$X(x) = \left[ \lambda_s^{p-1} \right]^{-1} \{\bar{Z}\}_0^T \{e^{\lambda_s x}\} \quad (14)$$

The equation  $|\bar{U}(\lambda_s)| = 0$  is the characteristic equation,  $\omega$  is the eigenvalue, and  $\{\bar{Z}\}_0$  is the eigenvector. The function  $X(x)$  is the mode shape.

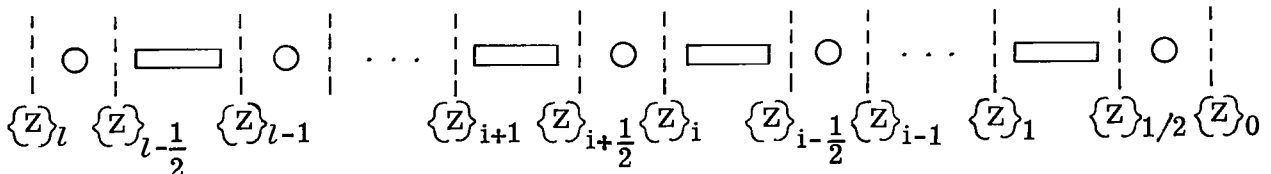
In the foregoing development, the beam was assumed to have uniform mass and stiffness along its length. If these quantities vary with position, the procedure is similar but the transfer matrix must be derived differently. The beam may be assumed piecewise uniform or the physical properties may be lumped at discrete positions on the beam. The lumping approach was applied to the Saturn V model and expressions used in that application are developed next.

#### Application to Saturn V Model

In the present analysis, the Saturn V model is visualized for mathematical purposes as a flexural beam with lumped mass, distributed stiffness, and attached branch systems. The manner in which the mass of the model is lumped is determined mainly by the actual physical arrangement. Examination of figure 2 shows that some of the mass is naturally lumped, or concentrated, at discrete points along the vehicle. This fact is reflected in the mathematical model shown in figure 8. The first-stage engines; the second- and

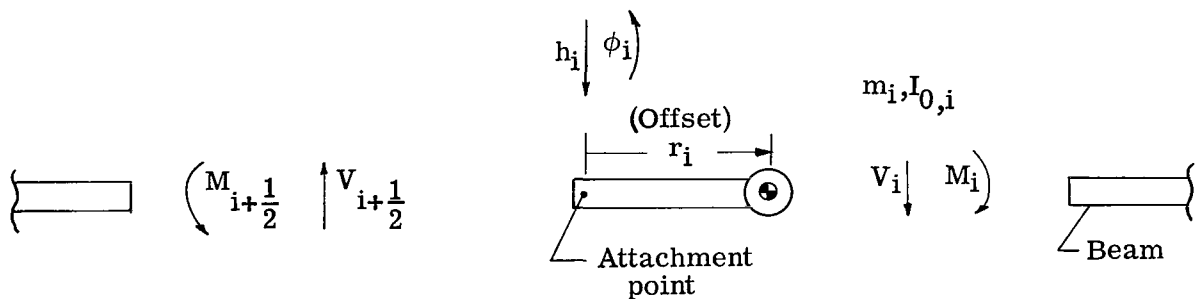
third-stage engines, thrust structures, and lox tanks; and the LM are attached as branched discrete systems. The first-stage engines and the LM are attached at the main beam through a rotational spring and a linear spring, respectively. The S-II engines and thrust structure are attached to the main beam at the thrust structure attachment station. The lox tank of the S-II stage is treated as a separate single-degree-of-freedom slosh mass attached to the main beam at the point where the lox tank is tangent to the outer shell. The S-IVB engine, thrust cone, and lox tank are treated as a multielement branched beam attached to the main beam at the S-IVB aft skirt attachment plane. The remainder of the model mass is concentrated at equally spaced points along the beam. The number of mass points used for each configuration is also shown in figure 8. Each mass point is allowed two degrees of freedom, translation and rotation.

The lumping thus idealizes the vehicle into a series of disconnected masses and massless elastic beams in the manner shown in sketch (a) so that the motion of the composite beam results from the elastic beams being described by the static beam equation, and the motion of the masses is governed by the dynamic equations for rigid bodies.



Sketch (a)

Transfer matrix for typical inertial element.— An inertial element is assumed to consist of a rigid body which applies an inertial force and moment to the point on the beam to which it is attached. The inertial forces result from the motion of the beam center line. In most instances the point of attachment will be taken to be the centroid. For the general case, however, the centroid will be assumed to be offset from the attachment point. The element is represented schematically in sketch (b) (small displacements are assumed).



Sketch (b)

Equilibrium of the external forces and moments results in two equations (positive values are indicated in sketch (b)):

$$V_{i+\frac{1}{2}} - V_i + m_i r_i \ddot{\phi} + m \ddot{h}_i = 0 \quad (15)$$

$$M_{i+\frac{1}{2}} - M_i - (m_i r_i^2 + I_{0,i}) \ddot{\phi} - m_i r_i \ddot{h}_i = 0 \quad (16)$$

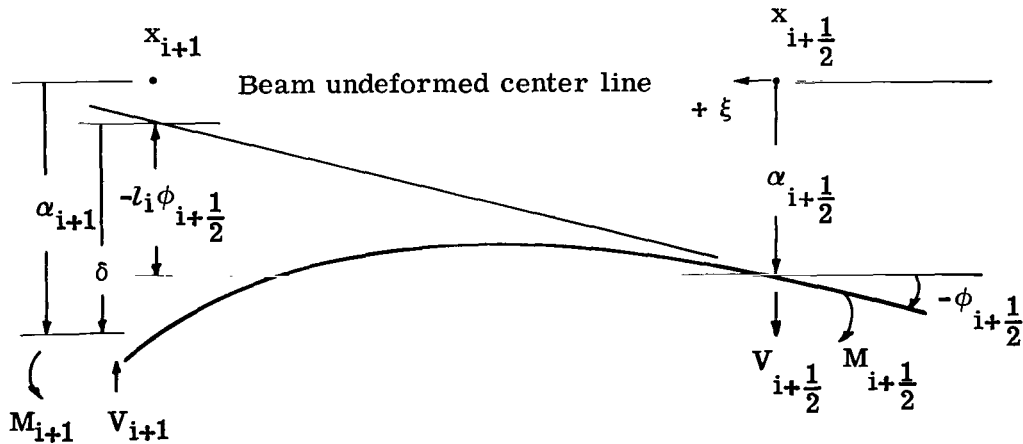
Simple harmonic motion is assumed. The equilibrium equations can be rewritten in matrix form as

$$\begin{Bmatrix} V \\ M \\ \phi \\ h \end{Bmatrix}_{i+\frac{1}{2}} = \begin{bmatrix} 1 & 0 & m_i r_i \omega^2 & m_i \omega^2 \\ 0 & 1 & -(m_i r_i^2 + I_{0,i}) \omega^2 & -m_i r_i \omega^2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} V \\ M \\ \phi \\ h \end{Bmatrix}_i \quad (17)$$

or

$$\{Z\}_{i+\frac{1}{2}} = [m(\omega)]_i \{Z\}_i$$

Transfer matrix for typical beam element.- The transfer matrix for the massless elastic beam element is derived from the static beam equations. A beam possessing bending flexibility, which may vary with  $x$ , is loaded as shown in sketch (c). The total



Sketch (c)

lateral deflection of the beam consists of two parts, that due to bending deformation and that due to shear deformation. The bending deflection at station  $i + 1$  is denoted  $\alpha_{i+1}$  and is equal to

$$\alpha_{i+1} = \alpha_{i+\frac{1}{2}} - \left( -l_i \phi_{i+\frac{1}{2}} \right) + \delta \quad (18)$$

where  $\delta$  is the tangential deviation and  $l_i = x_{i+1} - x_{i+\frac{1}{2}}$ . The bending slope at  $x_{i+1}$  is obtained from the beam equation

$$\frac{d\phi}{dx} = \frac{M}{EI}$$

by integrating between the  $x_{i+\frac{1}{2}}$  and  $x_{i+1}$ :

$$\phi_{i+1} - \phi_{i+\frac{1}{2}} = \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{M(\xi)d\xi}{E(\xi)I(\xi)} \quad (19)$$

The tangential deviation, as given in reference 21, is

$$\delta = \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{(l_i - \xi)M(\xi)d\xi}{E(\xi)I(\xi)} \quad (20)$$

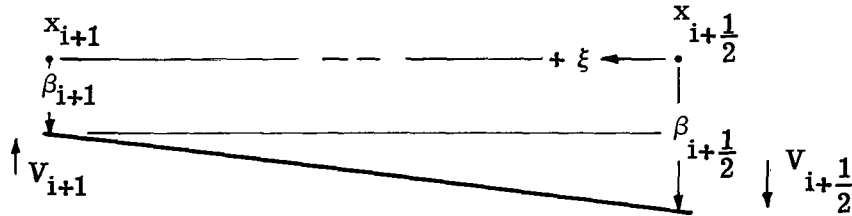
The moment is

$$M(\xi) = M_i + V_i \xi \quad (21)$$

Equations (18), (19), and (20) are combined and an expression for  $\alpha_{i+1}$  results:

$$\alpha_{i+1} = \alpha_{i+\frac{1}{2}} + l_i \phi_{i+\frac{1}{2}} + M_i \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{(l_i - \xi)d\xi}{E(\xi)I(\xi)} + V_i \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{\xi(l_i - \xi)d\xi}{E(\xi)I(\xi)} \quad (22)$$

In addition, if the beam can deform in shear, the total deflection will include both bending deflection  $\alpha_i$  and shear deflection  $\beta_i$ . As illustrated in sketch (d), the change



Sketch (d)

in shear deflection is, from reference 21,

$$\frac{d\beta}{d\xi} = - \frac{V}{K(\xi)A(\xi)G(\xi)} \quad (23)$$

Integrating between the limits  $x_{i+\frac{1}{2}}$  and  $x_{i+1}$  gives the shear deflection:

$$\beta_{i+1} - \beta_{i+\frac{1}{2}} = -V_i \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{d\xi}{K(\xi)A(\xi)G(\xi)} \quad (24)$$

The total deflection  $h_i$  is  $\alpha_i + \beta_i$ . Equations (22) and (24) are added to obtain

$$h_{i+1} = h_{i+\frac{1}{2}} + l_i \phi_{i+\frac{1}{2}} + M_i \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{(l_i - \xi)d\xi}{E(\xi)I(\xi)} + V_i \left[ \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{\xi(l_i - \xi)d\xi}{E(\xi)I(\xi)} - \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{d\xi}{K(\xi)A(\xi)G(\xi)} \right] \quad (25)$$

Equations (19), (21), and (25) may be rewritten in matrix form as

$$\begin{Bmatrix} V \\ M \\ \phi \\ h \end{Bmatrix}_{i+1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_i & 1 & 0 & 0 \\ c_1 & c_2 & 1 & 0 \\ c_3 & c_4 & l_i & 1 \end{bmatrix} \begin{Bmatrix} V \\ M \\ \phi \\ h \end{Bmatrix}_{i+\frac{1}{2}} \quad (26)$$

where

$$c_1 = \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{\xi d\xi}{E(\xi)I(\xi)}$$

$$c_2 = \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{d\xi}{E(\xi)I(\xi)}$$

$$c_3 = \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{\xi(l_i - \xi)d\xi}{E(\xi)I(\xi)} - \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{d\xi}{K(\xi)A(\xi)G(\xi)}$$

$$c_4 = \int_{x_{i+\frac{1}{2}}}^{x_{i+1}} \frac{(l_i - \xi)d\xi}{E(\xi)I(\xi)}$$

The equation may be written in abbreviated form as

$$\{Z\}_{i+1} = [e]_i \{Z\}_{i+\frac{1}{2}}$$

where  $[e]_i$  is the transfer matrix for the beam element.

Section properties.- The 1/10-scale Saturn V model was assumed to retain its cross-sectional geometry under deformation. As a result, the sectional area and area moment of inertia of the elastic beams were for the most part based simply on the following:

$$A_e = A_{\text{skin}} + A_{\text{str}} + A_D \quad (27)$$

$$I = \frac{A_e R^2}{2} \quad (28)$$

where

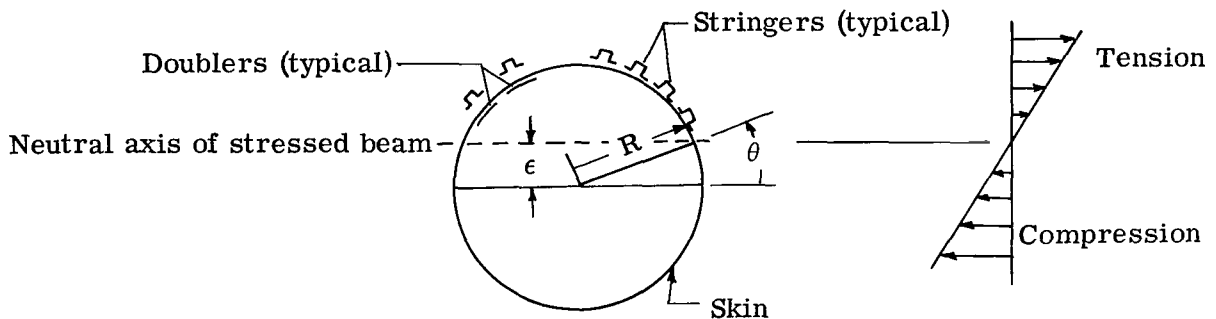
$A_e$             equivalent area

$A_{\text{str}}$         area of stringers

$A_D$            area of doublers

$R$             radius of shell

This procedure, in effect, distributes stringer and doubler area around the circumference and includes the total geometric area. The skirt and interstage sections of the S-II and S-IVB stages, however, employ very thin skin in their construction. For these areas,



Sketch (e)

shown in figure 3, the area moment of inertia, and thus the bending stiffness, was reduced on the assumption that skin in compression cannot support an axial load. The effect was incorporated in the analysis by using area-moment-of-inertia data based on  $\epsilon$ , a neutral-axis offset from the geometric center. (See sketch (e).) The magnitude of the offset reflects the assumption that the skin can support tension loads only, whereas the stringers can support both tension and compression. Section properties for the areas treated in this manner were obtained from the following equations:

$$I_{\text{skin}} = R^3 \tau \left\{ \frac{\pi}{2} \left[ 1 + \left( \frac{\epsilon}{R} \right)^2 \right] - \theta \left[ 1 - 2 \left( \frac{\epsilon}{R} \right)^2 \right] + \frac{1}{2} \sin \theta - 4 \left( \frac{\epsilon}{R} \right) \cos \theta \right\} \quad (29)$$

$$I_{\text{str}} = \frac{R^3 A_{\text{str}}}{2} \left[ 1 + 2 \left( \frac{\epsilon}{R} \right)^2 \right] \quad (30)$$

where

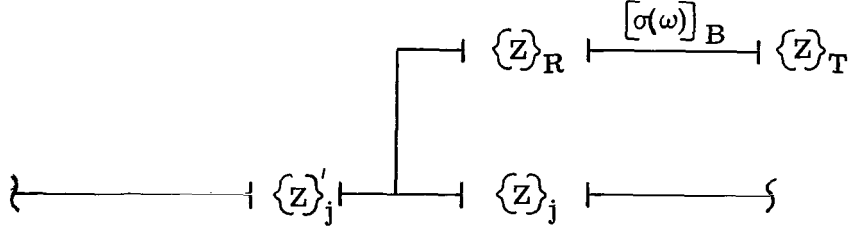
$$\theta = \text{ctn}^{-1} \left( \frac{1}{2} + \frac{A_{\text{str}}}{A_{\text{skin}}} \pi - \theta \right)$$

$$\frac{\epsilon}{R} = \sin \theta$$

Liquid slosh.- Liquid contained in the model tanks to simulate propellant is represented as a spring-mass system with characteristics calculated by the approximate method described in reference 22. The validity of a spring-mass analogy is substantiated in references 23 and 24. Derivations of the hydrodynamic properties may be found in references 24 and 25. In the present analysis, the amplitude of sloshing may be overestimated since the slosh baffles were not accounted for. Moreover, when the liquid level is near the top of the tank, the slosh mass is low and the frequency high. Consequently, the analytical representations of the S-II and S-IVB stages contain no slosh when these stages are at the 100-percent full condition. Slosh, when included, is represented by a one-mode spring-mass system. The remainder of the liquid mass is uniformly distributed along the tank as rigidly attached masses having no rotatory inertia.

Engine branches.- The first and second stages have five engines each, four outboard and one at the center line. The outboard engines, though gimbaled, are restrained from rotating by push rods designed to simulate the locked valve actuators. The center engine is held relatively rigid. Analytically, the outboard engines were combined into one single-degree-of-freedom system in which the mass is the total mass of four engines. A rotary restraining spring was sized so that the system frequency corresponds to a representative measured engine frequency. The S-IVB engine was treated similarly.

The fuel slosh masses and the engines are treated as branched systems. If the branched system is open – that is, if the branched system is attached to the main beam at a single point – its motion can be found from sketch (f). The column matrices



Sketch (f)

$\{Z\}_T$  and  $\{Z\}_R$  are the state vectors at the free end and the root of the branch. The column matrices  $\{Z\}_j$  and  $\{Z\}'_j$  are the state vectors on each side of the connection point. The procedure for developing the transfer matrix  $[\sigma(\omega)]_B$  from the elemental matrices  $[e]_i$  and  $[m]_i$  will be given later in this section. From conditions of equilibrium the internal loads to the left of the connection point are given by

$$\begin{Bmatrix} V \\ M \end{Bmatrix}'_j = \begin{Bmatrix} V \\ M \end{Bmatrix}_j + \begin{Bmatrix} V \\ M \end{Bmatrix}_R \quad (31)$$

Continuity of slope and deflection yield the relationship

$$\begin{Bmatrix} \phi \\ h \end{Bmatrix}'_j = \begin{Bmatrix} \phi \\ h \end{Bmatrix}_j = \begin{Bmatrix} \phi \\ h \end{Bmatrix}_R \quad (32)$$

The state vector  $\{Z\}_R$  is found from

$$\{Z\}_R = \begin{bmatrix} \sigma_{11}(\omega) & \sigma_{12}(\omega) \\ \sigma_{21}(\omega) & \sigma_{22}(\omega) \end{bmatrix} \begin{Bmatrix} V \\ M \\ \phi \\ h \end{Bmatrix}_T \quad (33)$$

For a free-ended branch,  $V_T = M_T = 0$ . It follows that

$$\left\{ \begin{array}{c} V \\ M \\ \hline \phi \\ h \end{array} \right\}_R = \left[ \begin{array}{c} \sigma_{12}(\omega) \\ \hline \sigma_{22}(\omega) \end{array} \right]_B \left\{ \begin{array}{c} \phi \\ h \end{array} \right\}_T \quad (34)$$

Equations (32) and (34) are combined with equation (31) to get an expression for the loads on the left of the connection point in terms of the deflections on the right:

$$\left\{ \begin{array}{c} V \\ M \end{array} \right\}_j' = \left\{ \begin{array}{c} V \\ M \end{array} \right\}_j + \left[ \sigma_{12}(\omega) \right]_R \left[ \sigma_{22}(\omega) \right]_B^{-1} \left\{ \begin{array}{c} \phi \\ h \end{array} \right\}_j \quad (35)$$

In practice the transfer matrices for the beam to the right of the joint are calculated, then the contribution from the branched system is added, and the transfer process is continued. Attention must be given to the sign of the loads to account for the direction in which the branch beam faces with respect to the main beam.

Boundary conditions.- When the frequency-dependent transfer matrices for all beam elements have been obtained, the transfer matrix for the entire beam is constructed by imposing conditions of continuity and equilibrium at the elemental interfaces. Consequently, the state vector at one end of the beam is associated with the state vector at the other end through the product of all the elemental transfer matrices; that is,

$$\{Z\}_l = [\sigma(\omega)] \{Z\}_0 \quad (36)$$

where

$$[\sigma(\omega)] = \prod_{i=0}^l [e]_{l-i} [m(\omega)]_{l-i}$$

The solution must satisfy prescribed boundary conditions at the two end points. The conditions are that any two of the elements of the state vector  $V$ ,  $M$ ,  $\phi$ , or  $h$  will be identically zero at either end. With these values included, equation (13) becomes

$$\{0\} = [\bar{\sigma}(\omega)] \{\bar{Z}\}_0 \quad (37)$$

where  $[\bar{\sigma}(\omega)]$  is the reduced transfer matrix obtained by application of the boundary conditions.

As an example of the manner in which the reduction is accomplished, a free-free beam for which  $V = M = 0$  at each end is considered. Equation (36) may be partitioned so that

$$\begin{Bmatrix} 0 \\ 0 \\ \phi \\ h \end{Bmatrix}_l = \begin{bmatrix} \sigma_{11}(\omega) & \sigma_{12}(\omega) \\ \hline \sigma_{21}(\omega) & \sigma_{22}(\omega) \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \phi \\ h \end{Bmatrix}_0 \quad (38)$$

From the first two equations of the system,

$$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = [\bar{\sigma}_{12}(\omega)] \begin{Bmatrix} \phi \\ h \end{Bmatrix}_0 \quad (39)$$

which is the desired determinantal equation. For other sets of boundary conditions, the transfer matrix will be reduced by a different partitioning.

In this analysis, the model was treated as a free-free beam. Therefore shear  $V$  and moment  $M$  were set equal to zero at each end. The assumption of zero shear and bending moment at the base of the model does not precisely represent the model in the test stand. However, the model suspension system was designed to impose very small shears and moments at the base. The mass of the suspension-system cradle was included in the analytical model to maintain agreement with the test specimen. The weight of the cables which constitute part of the suspension system was omitted.

A solution of equation (39) other than  $\{\bar{Z}\}_0 = \{0\}$  will exist if and only if the determinant

$$|\bar{\sigma}_{12}(\omega)| = 0 \quad (40)$$

The values of  $\omega$  for which the determinant vanishes are the eigenvalues  $\omega_c$ . The state vector components are then

$$\{\bar{Z}\}_0 = \text{adj}[\bar{\sigma}(\omega_c)] \{I\} \quad (41)$$

where  $\text{adj}[\bar{\sigma}(\omega_c)]$  is the adjoint of  $[\bar{\sigma}(\omega_c)]$  and  $\{I\}$  is a column matrix each element of which is unity. The state vector at any location  $i = j$  is then

$$\{Z\}_j = \prod_{i=0}^j [e]_{j-i} [m(\omega_c)]_{j-i} \{Z\}_0 \quad (42)$$

where  $\{Z\}_0$  now has elements  $0, 0, \phi_0$ , and  $h_0$ .

Values of  $\omega$  for which  $|\bar{\sigma}(\omega)| = 0$  are found numerically by observing the sign of  $|\bar{\sigma}(\omega)|$  as  $\omega$  is varied. A change of sign indicates either a zero crossing or an asymptote. An asymptote occurs for a value of  $\omega$  which corresponds to a fixed-base

resonant frequency of a branch beam – that is, an uncoupled mode. A zero crossing corresponds to a resonant frequency of the coupled system.

## RESULTS AND DISCUSSION

Data from analysis and tests of the lateral vibratory characteristics of the 1/10-scale Apollo/Saturn V model are presented in figures 9 to 25, tables 2 to 7, and the appendix. Complete data are shown for three propellant loadings (full, half-full, and empty) in each configuration. Tests were also conducted with other fractional propellant loadings (i.e., 25 percent and 75 percent). A summary of the measured resonant frequencies for each of the three configurations is presented in figure 9. The frequencies are plotted as a function of simulated propellant loading. Other preliminary tests were conducted early in the program which showed that such potential variables as tank ullage pressure, number of shakers, or shaker location had insufficient effect on the model frequencies and response shapes to warrant further consideration in the test program.

The data in figures 10 to 24 follow the same format for each of the three configurations examined: (1) a representative experimental frequency sweep taken in the yaw direction at the 100-percent propellant loading condition, which shows the accelerations of various points on the model structure plotted as a function of the frequency of the input force, and (2) composite plots of calculated and measured normalized response shapes at discrete frequencies. Tabulated on each response-shape plot are the values of the calculated frequency  $f_a$  and the measured frequencies in the pitch and yaw directions denoted  $f_p$  and  $f_y$ , respectively. As discussed previously, the analytical model is assumed to be radially symmetric; consequently, there is no distinction between the calculated pitch and yaw responses.

A check for orthogonality of the measured deflection shapes, including rigid-body translation and rigid-body rotation about the system center of gravity, was made by combining the measured deflection shapes with the calculated mass distributions in the following manner:

$$m_{gh} = \sum_{k=0}^n m(k)h_g(k)h_h(k) \quad (43)$$

and normalizing according to the relationship

$$M_{gh} = \frac{m_{gh}}{\sqrt{m_{gg}m_{hh}}} \quad (44)$$

Equivalent mass values  $\overline{M}_{gg}$  are then expressed in terms of the total system mass  $M_t$  as

$$\overline{M}_{gg} = \frac{m_{gg}}{M_t} \quad (45)$$

where  $h_g(k)$  and  $h_h(k)$  are the deflection shapes at station  $k$  in the  $g$ th and  $h$ th modes, and  $m(k)$  is the analytical concentrated mass at station  $k$  (including branch masses). If the deflection shapes are orthogonal, and if the calculated masses are accurate, then the generalized mass matrix having elements  $m_{gh}$  will be diagonal. The quality of the measured shapes is indicated by the magnitude of the off-diagonal elements of the matrix  $M_{gh}$  relative to unity. No precise criteria exist for good or bad orthogonality correlations and their relationships to errors in the measured deflection shapes. One NASA Space Vehicle Design Criteria document (ref. 26) arbitrarily sets the value of the off-diagonal element magnitude at 0.1 as a demarcation between acceptable and unacceptable shapes.

### Configuration I

Data from analysis and tests of configuration I (all stages plus payload) are presented in figures 10 to 13 and in table 2. Numerical values of experimental measurements are also given in the appendix. Figure 10 consists of curves of experimentally determined acceleration as a function of frequency for excitation in the yaw direction with S-IC propellant loading at 100 percent, corresponding to lift-off. Figures 11, 12, and 13 are composite plots of the normalized deflection shapes for the three weight conditions. The calculated mode shapes are indicated by solid curves while the measured deflections are indicated by circular and square symbols for pitch and yaw, respectively. The analytical frequency is denoted by  $f_a$  and the pitch and yaw frequencies from experiment are denoted by  $f_p$  and  $f_y$ , respectively. The various modes are plotted in order of ascending frequency and are denoted mode A, mode B, and so forth. This format and symbol notation are used for subsequent configurations also.

There is generally good agreement between the calculated and measured frequencies and response shapes for the first four bending modes of configuration I for three S-IC stage propellant loading conditions. Table 3 shows the correlation between the calculated frequencies and the measured pitch and yaw frequencies.

The analytically predicted frequencies are higher than either of the measured frequencies and, in general, the measured frequencies are higher in the pitch direction than in the yaw direction. These differences in measured frequency reflect model mass and stiffness asymmetries, some of which are representative of the full-scale structure and others of which may be due to fabrication imperfections. The measured, normalized, pitch and yaw deflection shapes are, however, nearly identical. Small, off-diagonal terms in the orthogonality matrices in table 2 indicate the absence of strong coupling or

TABLE 2.- GENERALIZED MASS MATRIX FOR CONFIGURATION I

| (a) Pitch direction    |     |        |        |       |        | (b) Yaw direction      |                     |            |     |     |        |        |        |        |            |                     |            |
|------------------------|-----|--------|--------|-------|--------|------------------------|---------------------|------------|-----|-----|--------|--------|--------|--------|------------|---------------------|------------|
| 100 percent propellant |     |        |        |       |        | 100 percent propellant |                     |            |     |     |        |        |        |        |            |                     |            |
|                        |     |        |        |       |        | Mode,<br>g             | $\overline{M}_{gg}$ | $f_g$ , Hz |     |     |        |        |        |        | Mode,<br>g | $\overline{M}_{gg}$ | $f_g$ , Hz |
| 1.0                    | 0   | 0.235  | -0.029 | 0.114 | -0.059 | 1                      | 1.0                 | 0          | 1.0 | 0   | -0.004 | -0.003 | -0.087 | -0.378 | 1          | 1.0                 | 0          |
|                        | 1.0 | -0.109 | 0.027  | 0.025 | 0.078  | 2                      | .028                | 0          |     | 1.0 | -0.085 | 0.028  | 0.040  | 0.127  | 2          | .0301               | 0          |
|                        |     | 1.0    | -0.027 | 0.395 | -0.071 | 3                      | .0059               | 9.1        |     |     | 1.0    | -0.129 | 0.050  | 0.340  | 3          | .00628              | 8.6        |
|                        |     |        | 1.0    | 0.165 | 0.109  | 4                      | .00423              | 16.0       |     |     |        | 1.0    | -0.158 | -0.054 | 4          | .00514              | 15.5       |
|                        |     |        |        | 1.0   | -0.210 | 5                      | .00224              | 24.8       |     |     |        |        | 1.0    | 0.169  | 5          | .00284              | 24.7       |
|                        |     |        |        |       | 1.0    | 6                      | .00206              | 30.4       |     |     |        |        |        | 1.0    | 6          | .00232              | 29.8       |

| 50 percent propellant |     |        |        |        |         | 50 percent propellant |        |      |     |     |        |        |        |        |   |        |      |
|-----------------------|-----|--------|--------|--------|---------|-----------------------|--------|------|-----|-----|--------|--------|--------|--------|---|--------|------|
| 1.0                   | 0   | 0.0422 | -0.077 | -0.217 | -0.0931 | 1                     | 1.0    | 0    | 1.0 | 0   | 0.073  | -0.044 | -0.072 | -0.075 | 1 | 1.0    | 0    |
|                       | 1.0 | -0.098 | 0.056  | -0.043 | 0.103   | 2                     | .0364  | 0    |     | 1.0 | -0.123 | 0.044  | 0.035  | 0.043  | 2 | .0388  | 0    |
|                       |     | 1.0    | -0.075 | 0.023  | -0.138  | 3                     | .0069  | 9.5  |     |     | 1.0    | -0.035 | -0.056 | -0.047 | 3 | .0074  | 9.1  |
|                       |     |        | 1.0    | -0.081 | -0.089  | 4                     | .0053  | 18.0 |     |     |        | 1.0    | 0.103  | 0.005  | 4 | .0055  | 17.9 |
|                       |     |        |        | 1.0    | -0.26   | 5                     | .00264 | 25.8 |     |     |        |        | 1.0    | 0.003  | 5 | .00227 | 25.9 |
|                       |     |        |        |        | 1.0     | 6                     | .00354 | 30.5 |     |     |        |        |        | 1.0    | 6 | .00352 | 30.0 |

| 0 percent propellant |     |        |        |        |  | 0 percent propellant |        |      |     |     |        |       |        |  |   |        |      |
|----------------------|-----|--------|--------|--------|--|----------------------|--------|------|-----|-----|--------|-------|--------|--|---|--------|------|
| 1.0                  | 0   | -0.026 | -0.072 | -0.055 |  | 1                    | 1.0    | 0    | 1.0 | 0   | -0.008 | 0.078 | -0.033 |  | 1 | 1.0    | 0    |
|                      | 1.0 | -0.037 | 0.007  | -0.12  |  | 2                    | .0439  | 0    |     | 1.0 | 0.007  | 0.003 | 0.001  |  | 2 | .0439  | 0    |
|                      |     | 1.0    | -0.054 | 0.089  |  | 3                    | .0149  | 10.0 |     |     | 1.0    | 0.035 | 0.077  |  | 3 | .0161  | 9.7  |
|                      |     |        | 1.0    | -0.151 |  | 4                    | .00103 | 19.3 |     |     |        | 1.0   | 0.088  |  | 4 | .00335 | 19.4 |
|                      |     |        |        | 1.0    |  | 5                    | .00058 | 29.0 |     |     |        |       | 1.0    |  | 5 | .00035 | 28.2 |

TABLE 3.- CONFIGURATION I FREQUENCY SUMMARY

| S-IC<br>propellant<br>load,<br>percent | $f_a$ ,<br>Hz | $\frac{f_a}{f_p}$ | $\frac{f_a}{f_y}$ | $\frac{f_p}{f_y}$ |
|--|---------------|-------------------|-------------------|-------------------|
| 100                                    | 9.15          | 1.00              | 1.06              | 1.05              |
|  | 17.30         | 1.08              | 1.12              | 1.03              |
|  | 26.16         | 1.05              | 1.06              | 1.00              |
|  | 33.16         | 1.09              | 1.11              | 1.02              |
| 50                                     | 9.82          | 1.03              | 1.08              | 1.04              |
|  | 19.07         | 1.06              | 1.06              | 1.00              |
|  | 27.30         | 1.05              | 1.05              | 1.00              |
|  | 34.25         | 1.12              | 1.14              | 1.02              |
| 0                                      | 10.29         | 1.03              | 1.06              | 1.03              |
|  | 20.31         | 1.05              | 1.05              | .99               |
|  | 28.93         | 1.00              | 1.02              | 1.02              |

branch mass resonances in the frequency range below 30 Hz. S-IC engine resonances occurred in the frequency range between 40 and 43 Hz, both analytically and experimentally. The generalized mass matrix of table 2 combines normalized measured deflection shapes and calculated idealized mass distribution. The off-diagonal elements may be either increased or reduced by variations in the calculated mass distribution, independent of the quality of the measured deflection shapes. In view of the lack of rigorous definition of the correlation between nonzero off-diagonal elements and the lack of orthogonality, results of the orthogonality checks can be used only to estimate the quality of the measured modes. The results are presented for this purpose and for the benefit of future investigators interested in the question of such correlations.

#### Configuration II

Data from analysis and tests of the model configuration II (S-II, S-IVB, and Apollo payload) are presented in figures 14 to 20 and table 4. Experimental data are also tabulated in the appendix. The format of presentation is identical to that employed for configuration I.

Figures 14 and 15 present the frequency-sweep data and deflection-shape plots for the configuration with 100 percent propellant and LES. The data show a set of three clearly defined, beam-type bending modes in the region between 10 and 60 Hz which agree quite well with the frequencies and mode shapes predicted by analysis. The orthogonality check (table 4) and the measured phase angles given in the appendix are also indicative of

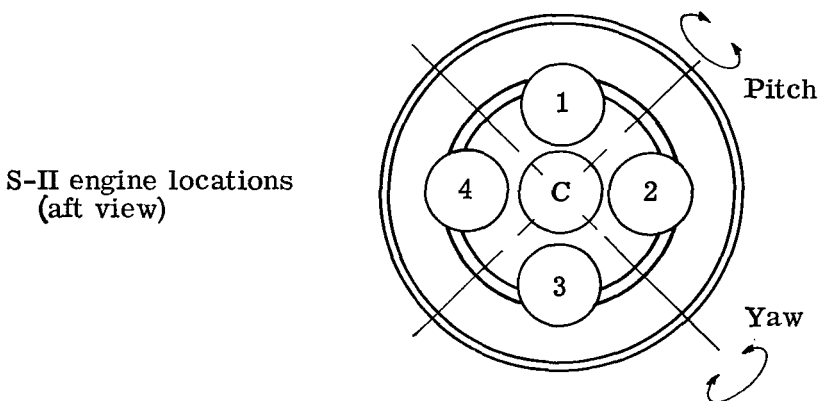
TABLE 4.- GENERALIZED MASS MATRIX FOR CONFIGURATION II

| (a) Pitch direction          |     |        |        |        |            |                |            | (b) Yaw direction            |     |        |         |         |            |                |            |
|------------------------------|-----|--------|--------|--------|------------|----------------|------------|------------------------------|-----|--------|---------|---------|------------|----------------|------------|
| 100 percent propellant + LES |     |        |        |        |            |                |            | 100 percent propellant + LES |     |        |         |         |            |                |            |
|                              |     |        |        |        | Mode,<br>g | $\bar{M}_{gg}$ | $f_g$ , Hz |                              |     |        |         |         | Mode,<br>g | $\bar{M}_{gg}$ | $f_g$ , Hz |
| 1.0                          | 0   | 0.059  | -0.133 | -0.063 | 1          | 1.0            | 0          | 1.0                          | 0   | 0.0009 | -0.0119 | -0.0196 | 1          | 1.0            | 0          |
|                              | 1.0 | -0.016 | 0.138  | 0.095  | 2          | .0438          | 0          |                              | 1.0 | 0.0080 | -0.017  | 0.0175  | 2          | .0438          | 0          |
|                              |     | 1.0    | 0.102  | -0.028 | 3          | .00969         | 14.0       |                              |     | 1.0    | 0.0917  | -0.0601 | 3          | .00846         | 14.0       |
|                              |     |        | 1.0    | 0.169  | 4          | .00462         | 27.8       |                              |     |        | 1.0     | 0.0807  | 4          | .00459         | 27.1       |
|                              |     |        |        | 1.0    | 5          | .269           | 55.8       |                              |     |        |         | 1.0     | 5          | .112           | 55.3       |
| 100 percent propellant       |     |        |        |        |            |                |            | 100 percent propellant       |     |        |         |         |            |                |            |
| 1.0                          | 0   | -0.001 | -0.009 | 0.081  | 1          | 1.0            | 0          | 1.0                          | 0   | -0.001 | -0.094  | -0.091  | 1          | 1.0            | 0          |
|                              | 1.0 | -0.011 | 0.005  | -0.095 | 2          | .057           | 0          |                              | 1.0 | 0.000  | -0.094  | -0.030  | 2          | .0159          | 0          |
|                              |     | 1.0    | 0.019  | 0.006  | 3          | .0312          | 16.3       |                              |     | 1.0    | 0.005   | 0.052   | 3          | .0316          | 16.5       |
|                              |     |        | 1.0    | 0.55   | 4          | .0629          | 52.6       |                              |     |        | 1.0     | 0.049   | 4          | .0793          | 50.4       |
|                              |     |        |        | 1.0    | 5          | .434           | 98.2       |                              |     |        |         | 1.0     | 5          | .115           | 87.2       |
| 50 percent propellant        |     |        |        |        |            |                |            | 50 percent propellant        |     |        |         |         |            |                |            |
| 1.0                          | 0   | 0.081  | -0.171 | -0.114 | 1          | 1.0            | 0          | 1.0                          | 0   | 0.139  | -0.071  | -0.043  | 1          | 1.0            | 0          |
|                              | 1.0 | -0.129 | -0.043 | -0.106 | 2          | .0745          | 0          |                              | 1.0 | -0.156 | -0.022  | -0.108  | 2          | .0208          | 0          |
|                              |     | 1.0    | 0.138  | 0.174  | 3          | .0540          | 17.0       |                              |     | 1.0    | 0.166   | 0.127   | 3          | .0561          | 17.4       |
|                              |     |        | 1.0    | 0.003  | 4          | .0668          | 53.0       |                              |     |        | 1.0     | -0.038  | 4          | .182           | 48.2       |
|                              |     |        |        | 1.0    | 5          | .192           | 100.0      |                              |     |        |         | 1.0     | 5          | .102           | 97.7       |
| 0 percent propellant         |     |        |        |        |            |                |            | 0 percent propellant         |     |        |         |         |            |                |            |
| 1.0                          | 0   | -0.035 | -0.142 | -0.431 | 1          | 1.0            | 0          | 1.0                          | 0   | 0.120  | -0.018  | -0.567  | 1          | 1.0            | 0          |
|                              | 1.0 | -0.048 | 0.139  | 0.283  | 2          | .070           | 0          |                              | 1.0 | 0.179  | 0.015   | 0.140   | 2          | .0191          | 0          |
|                              |     | 1.0    | -0.007 | 0.023  | 3          | .115           | 17.9       |                              |     | 1.0    | 0.134   | 0.186   | 3          | .067           | 18.1       |
|                              |     |        | 1.0    | -0.079 | 4          | .145           | 53.0       |                              |     |        | 1.0     | -0.042  | 4          | .573           | 48.3       |
|                              |     |        |        | 1.0    | 5          | .0353          | 110.9      |                              |     |        |         | 1.0     | 5          | .0889          | 100.3      |

the uncoupled nature of these modes. The resonance at  $f \approx 27$  Hz consists primarily of motion of the LES tower. Separate tests, not documented in this paper, showed that the CM-LES assembly, with the base of the CM fixed, has a first lateral bending frequency of 27.5 Hz.

Data from tests and analysis of configuration II with 100 percent propellant loading are shown in figures 16 and 18 and table 4. In this configuration, the model responds as a beam-type structure in only the lowest two modes. The correlation was fair between the analytical and experimental first-mode shapes and frequencies. The selection of the frequencies  $f_p = 52.6$  Hz and  $f_y = 50.4$  Hz as those of the second bending mode was somewhat arbitrary. As shown by the sweep plot of figure 16, significant vehicle response occurs throughout the frequency band between 45 and 60 Hz. The individual engine resonances shown in table 5 were identified experimentally from free-decay

TABLE 5.- DAMPED NATURAL FREQUENCIES OF UPPER-STAGE ENGINES



| Engine     | Damped natural frequency <sup>a</sup> with<br>propellant loading (S-II) of - |            |           |
|------------|--|------------|-----------|
|            | 100 percent  | 50 percent | 0 percent |
| SII-1      | 53   | 53.5       | 55        |
| SII-2      | 50   | 50.5       | 48        |
| SII-3      | 53-57  | 57         | 58        |
| SII-4      | 54   | 54-57      | 53-57     |
| SII center | 70.5   | 70.5       | 71        |
| S-IVB      | 51   | 50         | 51        |

<sup>a</sup>Engine frequencies were determined from observations of the decay time history resulting from a sudden removal of a sinusoidal excitation. The excitation was varied until a decay with minimum beat was obtained. The frequency at which an engine vibrated while decaying was then defined as its damped natural frequency.

oscillograms. The frequency at which an individual engine resonated and decayed most smoothly was considered to be its resonant frequency. The vehicle response shape for each of these resonances has the general appearance of a second beam-type mode. This situation existed for all weight conditions of configuration II, in both pitch and yaw directions. A typical set of these experimental resonant deflection shapes is shown in figure 18 for the 100 percent propellant condition with yaw excitation. Also shown are the individual engine response amplitudes.

The analytical resonance at  $f_a = 75.1$  Hz, shown in figure 17(c), is plotted with the yaw resonance at  $f_y = 87.2$  Hz purely on the basis of deflection-shape similarity. The measured acceleration amplitudes were extremely small (for example, the amplitude at  $x'/L' = 0.856$  was only 0.014g).

The pitch mode at  $f_p = 98.2$  Hz shown in figure 17(d) is not a vehicle bending mode; rather, it was found to be a radial or shell mode in the S-II liquid-hydrogen tank. The yaw resonance at  $f_y = 117.0$  Hz shown in figure 17(e) is primarily S-II engine motion, as may be seen in the sweep plot for this configuration (fig. 16).

Analytical and experimental dynamic response data for configuration II with 50 percent propellant are shown in figure 19 and in table 4. As in the 100-percent case, the lowest two modes have the classic beam-type shapes. The problem associated with isolation of the second mode from the coupled S-II engine-structure responses is again evident. Localized radial response patterns (discussed in a subsequent section) and component resonances are the chief contributors to the higher modes.

The data of figure 20 and table 4 for configuration II with 0 percent propellant, showed no significant deviations from the trends and patterns discussed for configuration II with 100 percent and 50 percent propellant. The figures show the analysis to be ineffective in predicting either frequency or deflection shape for other than the first two beam-type modes.

### Configuration III

Configuration III consists of the model S-IVB stage (including the aft interstage) and the Apollo payload (including the IU, the LM-SLA assembly, and the CM-SM assembly). Data from the analysis and tests of this configuration for pitch and yaw excitation with three simulated propellant loadings are shown in figures 21 to 24 and table 6. For 100 percent propellant (figs. 21 and 22) the data show three distinct response peaks below  $f = 115$  Hz. The pitch and yaw responses are similar in shape, but frequency differences of about 5 to 9 percent are shown. For both directions of excitation, the response peak at  $f \approx 40$  Hz appears to contain at least two resonant peaks. In previous tests the S-IVB engine was shown to resonate in the neighborhood of  $f \approx 50$  Hz. However, for this configuration the model response at  $f \approx 40$  Hz is a combined first beam mode

TABLE 6.- GENERALIZED MASS MATRIX FOR CONFIGURATION III

| (a) Pitch direction    |     |        |        |        |            |                     |            | (b) Yaw direction      |     |        |        |        |            |                     |            |       |
|------------------------|-----|--------|--------|--------|------------|---------------------|------------|------------------------|-----|--------|--------|--------|------------|---------------------|------------|-------|
| 100 percent propellant |     |        |        |        |            |                     |            | 100 percent propellant |     |        |        |        |            |                     |            |       |
|                        |     |        |        |        | Mode,<br>g | $\overline{M}_{gg}$ | $f_g$ , Hz |                        |     |        |        |        | Mode,<br>g | $\overline{M}_{gg}$ | $f_g$ , Hz |       |
| 1.0                    | 0   | -0.098 | 0.279  | -0.077 | 1          | 1.0                 | 0          | 1.0                    | 0   | -0.139 | 0.305  | -0.133 | 0.256      | 1                   | 1.0        | 0     |
|                        | 1.0 | 0.166  | -0.096 | -0.132 | 2          | .0682               | 0          |                        | 1.0 | 0.120  | -0.277 | 0.469  | 0.070      | 2                   | .00614     | 0     |
|                        |     | 1.0    | -0.190 | -0.031 | 3          | .0980               | 38.0       |                        |     | 1.0    | 0.203  | 0.256  | -0.434     | 3                   | .0104      | 40.4  |
|                        |     |        | 1.0    | 0.674  | 4          | .564                | 81.2       |                        |     |        | 1.0    | -0.106 | 0.636      | 4                   | .0151      | 76.3  |
|                        |     |        |        | 1.0    | 5          | .0307               | 99.0       |                        |     |        |        | 1.0    | -0.051     | 5                   | .00954     | 100.3 |
|                        |     |        |        |        |            |                     |            |                        |     |        |        | 1.0    |            | 6                   | .0103      | 114.7 |
| 50 percent propellant  |     |        |        |        |            |                     |            | 50 percent propellant  |     |        |        |        |            |                     |            |       |
| 1.0                    | 0   | -0.073 | 0.137  | 0.045  | 1          | 1.0                 | 0          | 1.0                    | 0   | -0.083 | 0.455  | 0.156  |            | 1                   | 1.0        | 0     |
|                        | 1.0 | 0.025  | -0.302 | 0.148  | 2          | .0896               | 0          |                        | 1.0 | 0.076  | -0.272 | -0.455 |            | 2                   | .00808     | 0     |
|                        |     | 1.0    | 0.164  | -0.659 | 3          | .104                | 40.4       |                        |     | 1.0    | 0.149  | -0.200 |            | 3                   | .269       | 43.3  |
|                        |     |        | 1.0    | 0.108  | 4          | .0548               | 98.4       |                        |     |        | 1.0    | 0.827  |            | 4                   | .600       | 86.1  |
|                        |     |        |        | 1.0    | 5          | .201                | 126.9      |                        |     |        |        | 1.0    |            | 5                   | .256       | 91.1  |
| 0 percent propellant   |     |        |        |        |            |                     |            | 0 percent propellant   |     |        |        |        |            |                     |            |       |
| 1.0                    | 0   | -0.048 | -0.431 | -0.066 | 1          | 1.0                 | 0          | 1.0                    | 0   | -0.016 | -0.177 | -0.10  |            | 1                   | 1.0        | 0     |
|                        | 1.0 | 0.059  | -0.018 | -0.214 | 2          | .101                | 0          |                        | 1.0 | 0.020  | -0.196 | 0.38   |            | 2                   | .00906     | 0     |
|                        |     | 1.0    | 0.513  | -0.084 | 3          | .206                | 41.2       |                        |     | 1.0    | -0.036 | -0.709 |            | 3                   | .176       | 43.5  |
|                        |     |        | 1.0    | 0.138  | 4          | .088                | 96.1       |                        |     |        | 1.0    | 0.243  |            | 4                   | .138       | 99.8  |
|                        |     |        |        | 1.0    | 5          | .196                | 103.3      |                        |     |        |        | 1.0    |            | 5                   | .595       | 134.2 |

and engine resonance. The pitch and yaw response peaks at  $f \approx 80$  Hz involve considerable LM motion not noted in the other configurations, the model center-line shape plotted in figure 22(b) being that of a second beam mode. At  $f \approx 100$  Hz the second beam mode shape, shown in figure 22(c), again appears with reduced LM response. Consequently, the analytical beam mode calculations were compared with the experimental data at  $f \approx 40$  Hz and  $f \approx 100$  Hz since the analysis does not, in its present form, permit lateral LM response. In the present analytical representation of the model, the LM is assumed to be rigidly attached at the model center line.

Considering the limitations which the mass distribution imposes on a beam-type analysis (to be discussed in a subsequent section) and the rigid LM assumptions previously mentioned, the analytical frequencies and mode shapes agree reasonably well with experiment in this weight condition.

The test and analysis data for the 50-percent weight condition, shown in figure 23 and table 6, exhibit many of the same characteristics as those discussed for the 100-percent weight condition. The first beam modes in the pitch and yaw directions agree reasonably well with those predicted by the analysis. The LM pitch-response mode at  $f = 86.1$  Hz (tabulated in the appendix but not plotted) is superimposed on a second beam-type center-line deflection shape.

The data from tests and analysis of the 0-percent propellant (burnout) condition for model configuration III are presented in figure 24 and table 6. The most significant departure from the trends for the other weight conditions lies in the fact that no LM pitch-response mode was measured in the region  $f \approx 80$  Hz. The correlations in both mode shape and frequency plotted in figure 24(b) are poor; the analytically predicted frequency is about 30 percent too high, and the analytical deflections in the S-IVB tank region,  $0.1 \leq x''/L'' \leq 0.5$ , are much lower than those measured. Considerable shell response was noted in the empty tanks, the IU, and the regions of the support cradle and aft interstage at frequencies above 90 Hz. Typical shell response patterns are illustrated in figure 25.

The one-dimensional mathematical model will not predict nonbeam resonances such as the shell responses illustrated in figure 25. This figure shows some of the typical measured radial response patterns from both configurations II and III. The measured nonbeam resonances are identified as S-II tank wall bulge, IU shell distortion, and support-cradle ring modes. Also noted on the figure are the configuration number, the simulated propellant loading, and the excitation frequency associated with each deflection pattern. Since the mathematical model does not permit two-dimensional motion, no calculations are available for comparison with shell and shell-like motion. From the preceding discussion, it would appear that either two- or three-dimensional representations would be required to predict a large number of lateral modes of the model. In most

preliminary analyses for control-system stability, wind loads, or similar applications, only the lower beam-type modes are required. Thus, a one-dimensional analysis of the type presented will be adequate to meet such requirements.

### Measured Damping Values

Representative damping values measured on the three configurations during the yaw-plane tests are given in table 7 together with associated resonant frequencies. The damping values are generally in the range of  $\frac{1}{2}$  to  $1\frac{1}{2}$  percent of critical damping. Similar results were obtained from pitch-plane tests.

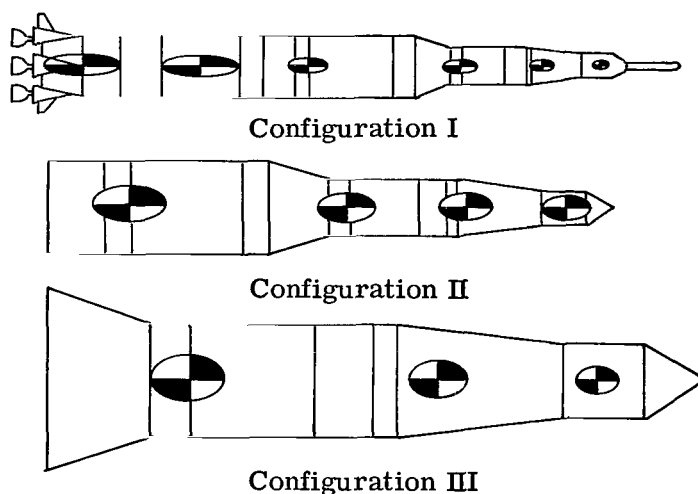
TABLE 7.- RESONANT FREQUENCIES AND DAMPING FOR EXCITATION IN YAW DIRECTION

| Configuration I |       |          |       |          |       | Configuration II |       |          |       |          |       |          |       | Configuration III |       |          |       |          |       |
|-----------------|-------|----------|-------|----------|-------|------------------|-------|----------|-------|----------|-------|----------|-------|-------------------|-------|----------|-------|----------|-------|
| 100%            |       | 50%      |       | 0%       |       | 100% + LES       |       | 100%     |       | 50%      |       | 0%       |       | 100%              |       | 50%      |       | 0%       |       |
| f,<br>Hz        | g     | f,<br>Hz | g     | f,<br>Hz | g     | f,<br>Hz         | g     | f,<br>Hz | g     | f,<br>Hz | g     | f,<br>Hz | g     | f,<br>Hz          | g     | f,<br>Hz | g     | f,<br>Hz | g     |
| 8.6             | 0.011 | 9.1      | 0.020 | 9.7      | 0.017 | 14.0             | 0.020 | 16.5     | 0.025 | 17.4     | 0.020 | 18.1     | 0.020 | 40.4              | 0.025 | 43.5     | 0.025 | 43.5     | 0.025 |
| 15.5            | .011  | 17.9     | .015  | 19.4     | .010  | 26.9             | .020  | ---      | ---   | ---      | ---   | ---      | ---   | 76.3              | .034  | 86.1     | ---   | ---      | ---   |
| 24.7            | .018  | 25.9     | .013  | ---      | ---   | 50.0             | ---   | 50.4     | .020  | 48.2     | .012  | 48.2     | ---   | 100.3             | .020  | 91.2     | .035  | 99.8     | .021  |
| 29.8            | .013  | 30.0     | .013  | 28.2     | .010  | ---              | ---   | 50.8     | .12   | 51.3     | .020  | 50.0     | ---   | 114.7             | ---   | ---      | ---   | ---      | ---   |
| 57.2            | .011  | 51.7     | .018  | ---      | ---   | 52.0             | ---   | 52.1     | .12   | 52.8     | .020  | ---      | ---   | ---               | ---   | 132.6    | .028  | 134.2    | .024  |
| 75.8            | .025  | 72.6     | .005  | ---      | ---   | 53.0             | ---   | 53.5     | ---   | 53.5     | .030  | 53.5     | ---   | ---               | ---   | 145.4    | .020  | ---      | ---   |
| ---             | ---   | 87.2     | .017  | ---      | ---   | 56.3             | .050  | 54.4     | .005  | 53.9     | ---   | 53.9     | ---   | ---               | ---   | ---      | ---   | ---      | ---   |
| ---             | ---   | ---      | ---   | ---      | ---   | 57.0             | ---   | 57.0     | ---   | 57.2     | ---   | 56.8     | ---   | ---               | ---   | ---      | ---   | ---      | ---   |
| ---             | ---   | ---      | ---   | ---      | ---   | 71.0             | ---   | 70.5     | .025  | 70.0     | ---   | 70.0     | ---   | ---               | ---   | ---      | ---   | ---      | ---   |
| ---             | ---   | ---      | ---   | ---      | ---   | ---              | ---   | 87.2     | ---   | ---      | ---   | ---      | ---   | ---               | ---   | ---      | ---   | ---      | ---   |
| ---             | ---   | ---      | ---   | ---      | ---   | 98.0             | ---   | 99.0     | ---   | 97.7     | ---   | 100.3    | .020  | ---               | ---   | ---      | ---   | ---      | ---   |
| ---             | ---   | ---      | ---   | ---      | ---   | 116.0            | ---   | 117.0    | ---   | 118.3    | .023  | 123.0    | ---   | ---               | ---   | ---      | ---   | ---      | ---   |

### Applicability of Mathematical Model

Large liquid masses concentrated over short distances along the length of the structure, particularly in the second and third stages, leave the analyst little control over the mathematical mass distribution. Sketch (g) illustrates this natural distribution.

For configuration I, six major mass concentrations exist. They are: S-IC engines and fuel, S-IC lox, S-II lox, S-IVB lox, LM-SLA, and CM-SM. With this type of natural mass lumping, a beam-type analysis would be expected to predict no more than five modes. For configuration II natural lumping produces four major mass concentrations; thus a beam analysis could be expected to predict no more than two modes. In the same context, the analysis would predict only one mode for configuration III. In actuality, the present analysis contains a greater number of mass points than are shown in the sketches; however, because of the relative magnitudes of these masses, the sketched configurations are realistic.



Sketch (g)

### Secondary Effects

In the mathematical representation, a number of assumptions were made which were expected to have a second-order effect on the basic beam response. Their importance can be assessed by comparing the measured results and the calculations made with and without the second-order effects.

Engine branches.— Engine resonances for the three stages fall in the range of 40 to 60 Hz. Since the first three beam modes of configuration I are less than 40 Hz, the effect of engine motion on these modes was not studied. In contrast to configuration I, the engine modes for configurations II and III have a decided effect on the interpretation of the beam-mode shapes since the second lateral mode of configuration II occurs in or near the S-II engine frequency band and the first lateral mode and the engine mode of configuration III are at proximate frequencies. The changes in modal frequency resulting from the assumption that the engines are rigidly attached to the model are illustrated in the table shown at right.

| Configuration | Calculated frequency, Hz, for — |              |
|---------------|---------------------------------|--------------|
|               | Flexible engine                 | Rigid engine |
| II-50%        | 18.64                           | 18.68        |
|               | 45.2                            | -----        |
|               | 51.71                           | -----        |
|               | 58.7                            | 57.2         |
|               | 87.2                            | 84.8         |
| III-100%      | 43.1                            | 43.1         |
|               | 45.3                            | -----        |
|               | 106.1                           | 105.8        |
|               | 135.3                           | 135.2        |

Liquid slosh.- No provision was made for experimentally measuring liquid motion. Therefore no direct correlation between experimental and analytical slosh-mass amplitude data was possible. Nevertheless, calculations were performed with and without sloshing to assess the magnitude of the effect. Typical results for configuration I with 100 percent propellant and configuration II with 50 percent propellant are given in the following table and show that the inclusion of slosh raises the frequencies of the beam-type modes from 5 to 15 percent over those obtained for the nonsloshing case. The magnitude of the change varies with the particular configuration.

| Configuration | Calculated frequency, Hz, with - |          |
|---------------|----------------------------------|----------|
|               | Slosh                            | No slosh |
| I-100%        | 20.1                             | 19.1     |
|               | 59.2                             | 58.0     |
|               | 86.6                             | 82.5     |
| II-50%        | 9.15                             | 8.92     |
|               | 17.30                            | 16.85    |
|               | 26.16                            | 25.92    |
|               | 33.16                            | 32.42    |

Ineffective skin.- In order to examine the influence of reduced stiffness due to the assumed inability of thin skin in certain areas to support compression loads, the model structure was analyzed with the assumption and without the assumption according to the methods discussed in the "Analysis" section. Typical results from these calculations are presented in the table at the left for the empty configuration II with rigid engines. The reduction in section stiffness lowers all system natural frequencies, the greatest change being a 2-percent reduction in the first mode.

| Calculated frequency, Hz, with - |                  |
|----------------------------------|------------------|
| Effective skin                   | Ineffective skin |
| 20.77                            | 19.45            |
| 53.84                            | 51.69            |
| 87.05                            | 85.31            |

Shear deformation.- All analytical results shown in this paper include shear deformation effects. One configuration I loading condition (100 percent) was analyzed with shear deformation omitted, and it was found that all calculated beam frequencies were increased by 10 to 20 percent.

## SUMMARY OF RESULTS

A 1/10-scale replica model of the Apollo/Saturn V launch vehicle has been tested and analyzed to determine its lateral vibration characteristics while suspended to simulate

free-free boundary restraints. Several propellant loadings were investigated for each of three model configurations simulating first-stage burn (configuration I), second-stage burn (configuration II), and third-stage burn (configuration III). Tests were conducted in both the pitch and yaw directions. A lumped-parameter vibration analysis employing the transfer matrix method was used to calculate the uncoupled natural lateral modes and frequencies of the system. Parameter studies were conducted to investigate analytically the effects of engine flexibility, liquid slosh, decreased stiffness due to ineffective skin, and shear deformation effects on the modal behavior of the system. Salient results of the combined analysis and experiment may be summarized as follows:

1. The model in the first-stage burn configuration responds to lateral excitation primarily as a beam and is therefore amenable to beam analysis. Correlation between predictions of the transfer matrix analysis and the measured response frequencies and shapes is good for model frequencies below 30 Hz.
2. The model in the second-stage burn configuration behaves as a beam only for the lowest two modes. The S-II engine branch-mass resonances which occur at frequencies proximate to the second beam mode tend to obscure the second beam mode. At higher frequencies, shell responses occur in the S-II liquid hydrogen tank and, at still higher frequencies, the payload shell responses predominate. The inclusion of shell modes and engine branch modes in the analysis would be required to predict adequately the behavior of this configuration at frequencies above the fundamental bending mode.
3. In the third-stage burn configuration, the lowest structural mode is a combination of a body beam mode and the S-IVB engine lateral mode. At higher frequencies, radial shell responses of the S-IVB tanks, instrument unit, and Saturn LM adapter preclude the existence of uncoupled beam-type structural modes. The one-dimensional analysis used in the present paper is inadequate for more than one mode of this configuration.
4. In the calculation of beam-bending stiffness properties, stiffness reduction due to loss of skin effectiveness for compression loads in sections having thin-gage skin-stringer construction should be considered. In the present analysis, however, the effect was minor.
5. The inclusion of liquid slosh in the analysis increased system resonant frequencies. This change degraded the correlation between tests and analysis.
6. Measured damping values generally are in the range of  $\frac{1}{2}$  to  $1\frac{1}{2}$  percent of critical damping.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., February 10, 1970.



## APPENDIX

### SUMMARY OF NORMALIZED RESONANT RESPONSES

Numerical data from both pitch and yaw tests of the 1/10-scale Apollo/Saturn V model are given in tables A-1 to A-10. Data are presented for three simulated propellant loadings in each of three model configurations. The data consist of the normalized amplitude of the fundamental harmonic, its phase angle with respect to the input force, and the magnitude of the normalization constant for accelerations measured by individual transducers at discrete resonant frequencies. Also given are the value of each resonant frequency, the peak magnitude of the sinusoidal input force, and the normalized model station at which the transducer was located.

The phase angle tabulated is the angle between the input force vector and the total response vector, measured in the counterclockwise direction. The acceleration amplitudes of the various transducers are normalized for the most part at the tip or forward station of the model. Low tip amplitudes in some modes, however, made alternate normalization stations desirable. The normalization stations are noted in footnotes on each table.

TABLE A-1.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION I  
WITH 100-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>x/L | Frequency = 9.1 Hz<br>Force = 19.1 N<br>a <sub>ref</sub> = 0.417 |                        | Frequency = 16.0 Hz<br>Force = 18.7 N<br>a <sub>ref</sub> = 0.961 |                        | Frequency = 24.8 Hz<br>Force = 18.9 N<br>a <sub>ref</sub> = 1.118 |                        | Frequency = 30.4 Hz<br>Force = 19.6 N<br>a <sub>ref</sub> = 0.611 |                        |     |
|--|--|------------------------|---|------------------------|---|------------------------|---|------------------------|-----|
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)  | (b)                    | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |     |
| 0.029                                  | 0.113  | 245                    | -0.086  | 75                     | 0.037   | 250                    | -0.024  | 59                     |     |
| .078                                   | .091   | 251                    | -.059   | 75                     | .021  | 255                    | -.020   | 67                     |     |
| .151                                   | .048   | 249                    | -.015   | 90                     | -.004   | 48                     | .005  | 164                    |     |
| .200                                   | .024   | 252                    | .016  | 247                    | -.022   | 62                     | .015  | 223                    |     |
| .346                                   | ----   | ---                    | -----   | ---                    | -----   | ---                    | -----   | ---                    |     |
| .412                                   | -.045  | 78                     | .011  | 262                    | .041  | 243                    | -.041   | 47                     |     |
| .432                                   | -.218  | 65                     | .002  | 24                     | .054  | 244                    | -.047   | 48                     |     |
| .565                                   | -.029  | 74                     | -.087   | 72                     | .038  | 247                    | .008  | 209                    |     |
| .648                                   | .007   | 215                    | -.163   | 74                     | -.011   | 48                     | .090  | 245                    |     |
| .734                                   | .105   | 244                    | -.172   | 74                     | -.090   | 63                     | .056  | 224                    |     |
| .772                                   | .177   | 245                    | -.129   | 74                     | -.117   | 65                     | -.006   | 20                     |     |
| .868                                   | .458   | 246                    | .137  | 251                    | -.102   | 68                     | -.195   | 45                     |     |
| .981                                   | 1.000  | 247                    | 1.000   | 253                    | 1.000   | 247                    | 1.000   | 224                    |     |
| S-1C engines                           |  |                        |   |                        |   |                        |   |                        |     |
| -0.024 {                               | Center   | 0.144                  | 124   | -0.140                 | 75  | -0.088                 | 110   | -0.090                 | 58  |
|  | 1  | .139                   | 237   | -.165                  | 75  | .130                   | 253   | -.195                  | 67  |
|  | 2  | .125                   | 123   | -.139                  | 74  | -.089                  | 108   | -.094                  | 62  |
|  | 3  | .137                   | 124   | -.144                  | 74  | -.098                  | 109   | -.118                  | 60  |
|  | 4  | .146                   | 124   | -.142                  | 75  | -.088                  | 109   | -.093                  | 59  |
| S-II engines                           |  |                        |   |                        |   |                        |   |                        |     |
| 0.369 {                                | Center   | -0.062                 | 56  | -0.029                 | 101   | -0.052                 | 117   | -0.077                 | 48  |
|  | 1  | -.067                  | 56  | -.032                  | 100   | -.061                  | 116   | -.096                  | 50  |
|  | 2  | -.067                  | 60  | -.044                  | 90  | -.265                  | 68  | .244                   | 155 |
|  | 3  | -.057                  | 57  | -.031                  | 99  | -.148                  | 90  | -.101                  | 51  |
|  | 4  | -.067                  | 55  | .031                   | 258   | .058                   | 244   | -.087                  | 51  |
| S-IVB engine                           |  |                        |   |                        |   |                        |   |                        |     |
| 0.600                                  | 0.053  | 240                    | -0.202  | 74                     | 0.021   | 263                    |   |                        |     |
| LM                                     |  |                        |   |                        |   |                        |   |                        |     |
| 0.775                                  | 0.170  | 250                    | -0.124  | 75                     | -0.129  | 66                     |   |                        |     |
| .807                                   | -.240  | 110                    | -.062   | 77                     | -.140   | 67                     |   |                        |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-1.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION I  
WITH 100-PERCENT PROPELLANT LOAD - Concluded

| (b) Yaw direction                      |  |                        |   |                        |   |                        |   |                        |     |
|--|--|------------------------|---|------------------------|---|------------------------|---|------------------------|-----|
| Normalized<br>model<br>station,<br>x/L | Frequency = 8.6 Hz<br>Force = 12.0 N<br>$a_{ref} = 0.377$<br>$g = 0.011$ |                        | Frequency = 15.5 Hz<br>Force = 9.8 Hz<br>$a_{ref} = 0.656$<br>$g = 0.011$ |                        | Frequency = 24.7 Hz<br>Force = 11.4 N<br>$a_{ref} = 0.485$<br>$g = 0.018$ |                        | Frequency = 29.8 Hz<br>Force = 11.3 N<br>$a_{ref} = 0.370$<br>$g = 0.013$ |                        |     |
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)  | (b)                    | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |     |
| 0.029                                  | 0.098  | 126                    | -0.091  | 151                    | 0.344   | 163                    | -0.024  | 161                    |     |
| .078                                   | .074   | 126                    | -.062   | 152                    | .023  | 170                    | -----   | ---                    |     |
| .151                                   | ----   | ---                    | -----   | ---                    | -.004   | 290                    | .005  | 296                    |     |
| .200                                   | ----   | ---                    | .018  | 323                    | -----   | ---                    | -----   | ---                    |     |
| .346                                   | ----   | ---                    | .046  | 334                    | .002  | 48                     | -----   | ---                    |     |
| .412                                   | ----   | ---                    | .015  | 344                    | .039  | 150                    | -----   | ---                    |     |
| .432                                   | -.064  | 306                    | -----   | ---                    | .049  | 152                    | -.054   | 149                    |     |
| .565                                   | ----   | ---                    | -.091   | 148                    | .039  | 153                    | -----   | ---                    |     |
| .648                                   | ----   | ---                    | -.175   | 150                    | -.008   | 276                    | .100  | 327                    |     |
| .734                                   | .101   | 121                    | -.166   | 150                    | -.074   | 331                    | .059  | 332                    |     |
| .772                                   | .180   | 121                    | -.139   | 148                    | -.109   | 333                    | ±.008   | 15                     |     |
| .868                                   | .456   | 124                    | .119  | 326                    | -.113   | 337                    | -.181   | 145                    |     |
| .981                                   | 1.000  | 126                    | 1.000   | 329                    | 1.000   | 153                    | 1.000   | 324                    |     |
| S-IC engines                           |  |                        |   |                        |   |                        |   |                        |     |
| -0.024 {                               | Center   | 0.138                  | 125   | -0.152                 | 155   | 0.082                  | 160   | -0.089                 | 160 |
|  | 1  | .133                   | 125   | -.145                  | 154   | .078                   | 163   | -.078                  | 158 |
|  | 2  | .133                   | 124   | -.149                  | 154   | .089                   | 167   | -.097                  | 160 |
|  | 3  | .138                   | 125   | -.149                  | 155   | .076                   | 161   | -.076                  | 163 |
|  | 4  | .138                   | 125   | -.151                  | 155   | .087                   | 164   | -.097                  | 160 |
| S-II engines                           |  |                        |   |                        |   |                        |   |                        |     |
| 0.369 {                                | Center   | ----                   | ---   | -----                  | ---   | 0.041                  | 144   | -0.068                 | 146 |
|  | 1  | ----                   | ---   | -----                  | ---   | -----                  | ---   | -----                  | --- |
|  | 2  | ----                   | ---   | -----                  | ---   | -----                  | ---   | -----                  | --- |
|  | 3  | ----                   | ---   | -----                  | ---   | -----                  | ---   | -----                  | --- |
|  | 4  | ----                   | ---   | -----                  | ---   | -----                  | ---   | -----                  | --- |
| S-IVB engine                           |  |                        |   |                        |   |                        |   |                        |     |
| 0.600                                  | 0.066  | 128                    | -0.221  | 151                    | 0.037   | 175                    | 0.235   | 327                    |     |
| LM                                     |  |                        |   |                        |   |                        |   |                        |     |
| 0.775                                  | 0.183  | 122                    | -0.133  | 154                    | -0.122  | 333                    | -0.008  | 83                     |     |
| .807                                   | .252   | 123                    | -.082   | 156                    | -.138   | 335                    | -.054   | 134                    |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-2.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION I  
WITH 50-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>x/L | Frequency = 9.5 Hz<br>Force = 14.7 N<br>a <sub>ref</sub> = 0.685 |                        | Frequency = 18.0 Hz<br>Force = 11.1 N<br>a <sub>ref</sub> = 1.030 |                        | Frequency = 25.8 Hz<br>Force = 11.6 N<br>a <sub>ref</sub> = 0.489 |                        | Frequency = 30.5 Hz<br>Force = 16.0 N<br>a <sub>ref</sub> = 0.811 |                        |     |
|--|--|------------------------|---|------------------------|---|------------------------|---|------------------------|-----|
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)  | (b)                    | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |     |
| 0.029                                  | 0.099  | 342                    | -0.097  | 333                    | 0.043   | 252                    | -0.031  | 154                    |     |
| .078                                   | .073   | 344                    | -.065   | 334                    | .024  | 270                    | -.017   | 164                    |     |
| .151                                   | .033   | 348                    | -.014   | 358                    | -.014   | 77                     | .011  | 297                    |     |
| .200                                   | .012   | 352                    | .019  | 144                    | -.037   | 71                     | .025  | 318                    |     |
| .346                                   | -----  | ---                    | -----   | ---                    | -----   | ---                    | -----   | ---                    |     |
| .412                                   | -.069  | 159                    | .045  | 153                    | .029  | 245                    | -.042   | 136                    |     |
| .432                                   | -.072  | 160                    | .036  | 153                    | .041  | 247                    | -.050   | 137                    |     |
| .565                                   | -.063  | 163                    | -.052   | 331                    | .039  | 249                    | .009  | 292                    |     |
| .648                                   | -.034  | 169                    | -.142   | 332                    | .004  | 270                    | .094  | 313                    |     |
| .734                                   | .069   | 336                    | -.180   | 333                    | -.072   | 68                     | .057  | 313                    |     |
| .772                                   | .130   | 340                    | -.147   | 334                    | -.104   | 70                     | -.004   | 90                     |     |
| .868                                   | .439   | 341                    | .072  | 149                    | -.119   | 77                     | -.197   | 133                    |     |
| .981                                   | 1.000  | 341                    | 1.000   | 152                    | 1.000   | 251                    | 1.000   | 313                    |     |
| S-IC engines                           |  |                        |   |                        |   |                        |   |                        |     |
| -0.024                                 | Center   | 0.140                  | 17  | -0.177                 | 27  | -0.127                 | 99  | 0.117                  | 332 |
|  | 1  | .152                   | 343   | -.212                  | 333   | .184                   | 264   | -.274                  | 161 |
|  | 2  | .136                   | 17  | -.176                  | 28  | -.127                  | 97  | .127                   | 335 |
|  | 3  | .136                   | 18  | -.184                  | 28  | -.143                  | 98  | .158                   | 334 |
|  | 4  | .140                   | 17  | -.178                  | 27  | -.125                  | 99  | .120                   | 332 |
| S-II engines                           |  |                        |   |                        |   |                        |   |                        |     |
| 0.369                                  | Center   | -0.066                 | 160   | 0.067                  | 153   | -0.033                 | 108   | 0.070                  | 313 |
|  | 1  | -.067                  | 159   | .072                   | 153   | -.037                  | 107   | .088                   | 317 |
|  | 2  | -.082                  | 162   | .164                   | 166   | .215                   | 14  | -.249                  | 68  |
|  | 3  | -.073                  | 162   | .087                   | 163   | -.041                  | 102   | .100                   | 319 |
|  | 4  | -.067                  | 159   | .071                   | 153   | .035                   | 256   | -.079                  | 136 |
| S-IVB engine                           |  |                        |   |                        |   |                        |   |                        |     |
| 0.600                                  | -0.085   | 163                    | -0.180  | 334                    | 0.049   | 270                    | 0.245   | 321                    |     |
| LM                                     |  |                        |   |                        |   |                        |   |                        |     |
| 0.775                                  | 0.130  | 340                    | -0.147  | 334                    | -0.110  | 79                     | -0.010  | 115                    |     |
| .807                                   | .142   | 340                    | -.151   | 333                    | -.127   | 79                     | .068  | 310                    |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-2.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION I  
WITH 50-PERCENT PROPELLANT LOAD - Concluded

(b) Yaw direction

| Normalized<br>model<br>station,<br>x/L | Frequency = 9.1 Hz<br>Force = 12.4 N<br>a <sub>ref</sub> = 0.531<br>g = 0.020 |                        | Frequency = 17.9 Hz<br>Force = 13.7 N<br>a <sub>ref</sub> = 0.975<br>g = 0.015 |                        | Frequency = 25.9 Hz<br>Force = 12.9 N<br>a <sub>ref</sub> = 1.210<br>g = 0.013 |                        | Frequency = 30.0 Hz<br>Force = 13.1 N<br>a <sub>ref</sub> = 0.649<br>g = 0.013 |                        |     |
|--|---|------------------------|--|------------------------|--|------------------------|--|------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg |     |
|  | (a)   | (b)                    | (a)  | (b)                    | (a)  | (b)                    | (a)  | (b)                    |     |
| 0.029                                  | 0.096   | 93                     | -0.103   | 150                    | 0.050  | 302                    | -0.039   | 258                    |     |
| .078                                   | .072  | 93                     | -.070  | 150                    | .031   | 303                    | -.020  | 265                    |     |
| .151                                   | .026  | 98                     | -----  | ---                    | -----  | ---                    | .012   | 54                     |     |
| .200                                   | -----   | ---                    | .021   | 325                    | -----  | ---                    | .034   | 65                     |     |
| .346                                   | -----   | ---                    | .065   | 320                    | -----  | ---                    | -----  | ---                    |     |
| .412                                   | -.068   | 270                    | .052   | 322                    | .015   | 289                    | -.052  | 246                    |     |
| .432                                   | -.072   | 271                    | .041   | 333                    | .025   | 292                    | -.062  | 245                    |     |
| .565                                   | -.064   | 274                    | -.051  | 144                    | .039   | 299                    | -----  | ---                    |     |
| .648                                   | -.041   | 278                    | -.146  | 148                    | -----  | ---                    | .106   | 63                     |     |
| .734                                   | .058  | 87                     | -.170  | 148                    | -.039  | 112                    | .068   | 66                     |     |
| .772                                   | .126  | 90                     | -.159  | 150                    | -.080  | 116                    | .014   | 77                     |     |
| .868                                   | .420  | 92                     | .046   | 325                    | -.135  | 120                    | -.185  | 243                    |     |
| .981                                   | 1.000   | 94                     | 1.000  | 329                    | 1.000  | 298                    | 1.000  | 61                     |     |
| S-IC engines                           |   |                        |  |                        |  |                        |  |                        |     |
| -0.024                                 | Center  | 0.136                  | 94   | -0.195                 | 141  | 0.131                  | 302  | -0.139                 | 257 |
|  | 1   | .128                   | 94   | -.184                  | 140  | .119                   | 302  | -.125                  | 256 |
|  | 2   | .128                   | 93   | -.189                  | 140  | .131                   | 303  | -.160                  | 254 |
|  | 3   | .136                   | 95   | -.192                  | 141  | -.122                  | 122  | -.126                  | 255 |
|  | 4   | .132                   | 94   | -.195                  | 140  | -.136                  | 122  | -.157                  | 256 |
| S-II engines                           |   |                        |  |                        |  |                        |  |                        |     |
| 0.369                                  | Center  | 0.064                  | 266  | 0.075                  | 316  | 0.116                  | 279  | -0.072                 | 240 |
|  | 1   | -----                  | ---  | -----                  | ---  | -----                  | ---  | -----                  | --- |
|  | 2   | -----                  | ---  | -----                  | ---  | -----                  | ---  | -----                  | --- |
|  | 3   | -----                  | ---  | -----                  | ---  | -----                  | ---  | -----                  | --- |
|  | 4   | -----                  | ---  | -----                  | ---  | -----                  | ---  | -----                  | --- |
| S-IVB engine                           |   |                        |  |                        |  |                        |  |                        |     |
| 0.600                                  | -0.096  | 274                    | -0.198   | 139                    | -0.901   | 124                    | -0.285   | 244                    |     |
| LM                                     |   |                        |  |                        |  |                        |  |                        |     |
| 0.775                                  | 0.137   | 90                     | -0.168   | 140                    | -0.917   | 118                    | 0.009  | 103                    |     |
| .807                                   | .209  | 92                     | -.129  | 140                    | -.117  | 119                    | -.046  | 236                    |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-3.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION I  
WITH 0-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>x/L | Frequency = 10.0 Hz<br>Force = 19.1 N<br>a <sub>ref</sub> = 0.717 |                        | Frequency = 19.3 Hz<br>Force = 13.8 N<br>a <sub>ref</sub> = 1.198 |                        | Frequency = 29.0 Hz<br>Force = 14.2 N<br>a <sub>ref</sub> = 0.690 |                        |     |
|--|---|------------------------|---|------------------------|---|------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |     |
| 0.029                                  | 0.146   | 36                     | -0.077  | 354                    | 0.023   | 8                      |     |
| .078                                   | .119  | 36                     | -.052   | 357                    | .016  | 38                     |     |
| .151                                   | .075  | 36                     | -.013   | 19                     | -.014   | 90                     |     |
| .200                                   | .049  | 38                     | .013  | 138                    | -.017   | 104                    |     |
| .346                                   | -.006   | 270                    | .008  | 235                    | -----   | ---                    |     |
| .412                                   | -.049   | 212                    | .060  | 166                    | -.019   | 139                    |     |
| .432                                   | -.054   | 213                    | .057  | 167                    | -.014   | 141                    |     |
| .565                                   | -.063   | 216                    | -.027   | 346                    | .028  | 317                    |     |
| .648                                   | -.047   | 218                    | -.121   | 346                    | .068  | 320                    |     |
| .734                                   | .039  | 24                     | -.169   | 347                    | .014  | 336                    |     |
| .772                                   | .102  | 27                     | -.160   | 348                    | -.039   | 131                    |     |
| .868                                   | .399  | 31                     | .026  | 156                    | -.071   | 138                    |     |
| .981                                   | 1.000   | 32                     | 1.000   | 167                    | 1.000   | 318                    |     |
| S-IC engines                           |   |                        |   |                        |   |                        |     |
| -0.024                                 | Center  | 0.194                  | 37  | -0.148                 | 6   | 0.070                  | 8   |
|  | 1   | .212                   | 37  | -.181                  | 354   | .129                   | 12  |
|  | 2   | .192                   | 37  | -.146                  | 6   | .072                   | 8   |
|  | 3   | .198                   | 36  | -.159                  | 6   | .088                   | 8   |
|  | 4   | .195                   | 38  | -.149                  | 6   | .071                   | 8   |
| S-II engines                           |   |                        |   |                        |   |                        |     |
| 0.369                                  | Center  | -0.046                 | 148   | 0.086                  | 166   | -0.038                 | 143 |
|  | 1   | -.043                  | 151   | .094                   | 167   | -.045                  | 145 |
|  | 2   | -.045                  | 153   | .204                   | 179   | -.151                  | 76  |
|  | 3   | -.051                  | 148   | .146                   | 176   | -.054                  | 148 |
|  | 4   | -.046                  | 207   | .091                   | 167   | -.043                  | 144 |
| S-IVB engine                           |   |                        |   |                        |   |                        |     |
| 0.600                                  | -0.092  | 215                    | -0.146  | 350                    | 0.256   | 324                    |     |
| LM                                     |   |                        |   |                        |   |                        |     |
| 0.775                                  | 0.107   | 30                     | -0.165  | 346                    | -0.049  | 140                    |     |
| .807                                   | .181  | 31                     | -.129   | 12                     | -.094   | 143                    |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-3.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION I  
WITH 0-PERCENT PROPELLANT LOAD - Concluded

(b) Yaw direction

| Normalized<br>model<br>station,<br>x/L | Frequency = 9.7 Hz<br>Force = 17.8 N<br>a <sub>ref</sub> = 0.482<br>g = 0.017 |                        | Frequency = 19.4 Hz<br>Force = 10.4 N<br>a <sub>ref</sub> = 0.892<br>g = 0.010 |                        | Frequency = 28.2 Hz<br>Force = 10.2 N<br>a <sub>ref</sub> = 0.572<br>g = 0.010 |                        |     |
|--|---|------------------------|--|------------------------|--|------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg |     |
|  | (a)   | (b)                    | (a)  | (b)                    | (a)  | (b)                    |     |
| 0.029                                  | 0.149   | 154                    | -0.075   | 151                    | 0.023  | 190                    |     |
| .078                                   | .118  | 155                    | -.049  | 152                    | .016   | 204                    |     |
| .151                                   | .056  | 159                    | -.009  | 176                    | -.009  | 263                    |     |
| .200                                   | .048  | 162                    | .016   | 306                    | -----  | ---                    |     |
| .346                                   | -.031   | 298                    | .048   | 322                    | -.023  | 325                    |     |
| .412                                   | -.058   | 314                    | .061   | 324                    | -.017  | 332                    |     |
| .432                                   | -.058   | 325                    | .058   | 324                    | -.016  | 335                    |     |
| .565                                   | -.064   | 324                    | -.028  | 140                    | .030   | 148                    |     |
| .648                                   | -.054   | 336                    | -----  | ---                    | .070   | 150                    |     |
| .734                                   | .044  | 145                    | -.159  | 144                    | .012   | 167                    |     |
| .772                                   | .098  | 160                    | -.156  | 144                    | -.038  | 323                    |     |
| .868                                   | .398  | 151                    | .012   | 310                    | -.163  | 329                    |     |
| .981                                   | 1.000   | 153                    | 1.000  | 324                    | 1.000  | 148                    |     |
| S-IC engines                           |   |                        |  |                        |  |                        |     |
| -0.024                                 | Center  | 0.216                  | 147  | -0.142                 | 149  | 0.066                  | 184 |
|  | 1   | .212                   | 142  | -.135                  | 148  | .061                   | 182 |
|  | 2   | .210                   | 140  | -.139                  | 148  | .072                   | 184 |
|  | 3   | .214                   | 148  | -.138                  | 149  | .061                   | 184 |
|  | 4   | .216                   | 144  | -.142                  | 148  | .073                   | 183 |
| S-II engines                           |   |                        |  |                        |  |                        |     |
| 0.369                                  | Center  | -0.050                 | 307  | 0.081                  | 319  | -----                  | --- |
|  | 1   | -----                  | ---  | -----                  | ---  | -----                  | --- |
|  | 2   | -----                  | ---  | -----                  | ---  | -----                  | --- |
|  | 3   | -----                  | ---  | -----                  | ---  | -----                  | --- |
|  | 4   | -----                  | ---  | -----                  | ---  | -----                  | --- |
| S-IVB engine                           |   |                        |  |                        |  |                        |     |
| 0.600                                  | 0.114   | 143                    | 0.298  | 322                    | 0.175  | 150                    |     |
| LM                                     |   |                        |  |                        |  |                        |     |
| 0.775                                  | 0.129   | 137                    | -0.157   | 143                    | -0.049   | 325                    |     |
| .807                                   | .210  | 141                    | -.131  | 143                    | -.089  | 327                    |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-4.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 100-PERCENT PROPELLANT LOAD PLUS  
LAUNCH ESCAPE SYSTEM

(a) Pitch direction

| Normalized<br>model<br>station,<br>x'/L' | Frequency = 14.0 Hz<br>Force = 16.9 N<br>a <sub>Ref</sub> = 0.597 |                        | Frequency = 27.8 Hz<br>Force = 16.0 N<br>a <sub>ref</sub> = 0.799 |                        | Frequency = 55.8 Hz<br>Force = 15.6 N<br>a <sub>ref</sub> = 0.268 |                        |     |
|--|---|------------------------|---|------------------------|---|------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |     |
| 0.002                                    | 0.085   | 338                    | -0.031  | 303                    | 0.026   | 288                    |     |
| .077                                     | .047  | 344                    | -.015   | 310                    | .011  | 307                    |     |
| .109                                     | .032  | 345                    | -.010   | 323                    | .007  | 13                     |     |
| .190                                     | -.003   | 155                    | .009  | 74                     | -.019   | 74                     |     |
| .244                                     | -.030   | 163                    | .019  | 97                     | -.026   | 90                     |     |
| .317                                     | -.062   | 164                    | .028  | 103                    | -.022   | 104                    |     |
| .371                                     | -.089   | 155                    | .038  | 107                    | -.030   | 114                    |     |
| .448                                     | -.122   | 163                    | .048  | 110                    | -.026   | 133                    |     |
| .581                                     | -.101   | 165                    | -.014   | 270                    | .067  | 292                    |     |
| .641                                     | -.057   | 167                    | -.061   | 285                    | .101  | 295                    |     |
| .792                                     | .223  | 341                    | -.160   | 289                    | -.041   | 125                    |     |
| .970                                     | 1.000   | 336                    | 1.000   | 110                    | 1.000   | 300                    |     |
| S-II engines                             |   |                        |   |                        |   |                        |     |
| 0.009 {                                  | Center  | 0.080                  | 20  | 0.426                  | 32  | -0.276                 | 69  |
|  | 1   | .084                   | 342   | .144                   | 323   | -.172                  | 136 |
|  | 2   | .084                   | 338   | .180                   | 325   | -.597                  | 141 |
|  | 3   | .094                   | 22  | .215                   | 35  | -1.321                 | 60  |
|  | 4   | .082                   | 340   | .084                   | 318   | -.601                  | 58  |
| S-IVB engine                             |   |                        |   |                        |   |                        |     |
| 0.372                                    | -0.149  | 156                    | 0.217   | 130                    | 0.672   | 301                    |     |
| LM                                       |   |                        |   |                        |   |                        |     |
| 0.647                                    | -0.054  | 159                    | -0.073  | 287                    | 0.127   | 300                    |     |
| .696                                     | .012  | 44                     | .109  | 71                     | -.116   | 59                     |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-4.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 100-PERCENT PROPELLANT LOAD PLUS  
LAUNCH ESCAPE SYSTEM - Concluded

(b) Yaw direction

| Normalized<br>model<br>station,<br>$x'/L'$ | Frequency = 14.0 Hz<br>Force = 16.4 N<br>$a_{\text{ref}} = 1.040$<br>$g = 0.020$ |                        | Frequency = 26.9 Hz<br>Force = 16.9 N<br>$a_{\text{ref}} = 1.190$<br>$g = 0.020$ |                        | Frequency = 56.3 Hz<br>Force = 14.2 N<br>$a_{\text{ref}} = 0.066$<br>$g = 0.050$ |                        |    |
|--|--|------------------------|--|------------------------|--|------------------------|----|
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg |    |
|  | (a)  | (b)                    | (a)  | (b)                    | (a)  | (b)                    |    |
| 0.002                                      | 0.074  | 320                    | -0.025   | 298                    | 0.151  | 265                    |    |
| .077                                       | .041   | 315                    | -.013  | 295                    | .076   | 259                    |    |
| .109                                       | .028   | 313                    | -.007  | 291                    | .030   | 231                    |    |
| .190                                       | -.009  | 160                    | .007   | 126                    | -.076  | 90                     |    |
| .244                                       | -.033  | 144                    | .018   | 122                    | -.151  | 90                     |    |
| .317                                       | -.065  | 143                    | .029   | 121                    | -.182  | 80                     |    |
| .371                                       | -.092  | 152                    | .036   | 119                    | -.197  | 67                     |    |
| .448                                       | -.130  | 141                    | .047   | 120                    | -.167  | 130                    |    |
| .496                                       | -.123  | 155                    | .029   | 128                    | .167   | 198                    |    |
| .581                                       | -.091  | 143                    | -.019  | 277                    | .333   | 264                    |    |
| .641                                       | -.046  | 134                    | -.066  | 290                    | .485   | 270                    |    |
| .696                                       | .030   | 317                    | -.114  | 293                    | .439   | 274                    |    |
| .763                                       | .145   | 329                    | -.162  | 295                    | .061   | 270                    |    |
| .827                                       | .310   | 318                    | -.127  | 298                    | -.485  | 100                    |    |
| .970                                       | 1.000  | 330                    | 1.000  | 113                    | 1.000  | 277                    |    |
| S-II engines                               |  |                        |  |                        |  |                        |    |
| 0.009                                      | Center   | 0.072                  | 23   | 0.029                  | 63   | 0.591                  | 97 |
|  | 1  | .076                   | 337  | -.036                  | 297  | 2.864                  | 84 |
|  | 2  | .077                   | 338  | -.039                  | 298  | 1.136                  | 79 |
|  | 3  | .074                   | 23   | .035                   | 65   | 6.212                  | 84 |
|  | 4  | .076                   | 23   | .035                   | 64   | 1.606                  | 54 |
| S-IVB engine                               |  |                        |  |                        |  |                        |    |
| 0.372                                      | -0.167   | 200                    | -0.127   | 239                    | -2.939   | 305                    |    |
| LM   |  |                        |  |                        |  |                        |    |
| 0.647                                      | 0.040  | 340                    | 0.077  | 112                    | 0.636  | 94                     |    |
| .696                                       | -.019  | 133                    | .112   | 113                    | .636   | 96                     |    |

<sup>a</sup>Normalized with respect to amplitude at forward end of LES.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-5.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 100-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>x'/L' | Frequency = 16.4 Hz<br>Force = 12.4 N<br>a <sub>ref</sub> = 0.520 |                        | Frequency = 52.6 Hz<br>Force = 17.8 N<br>a <sub>ref</sub> = 0.114 |                        | Frequency = 98.2 Hz<br>Force = 16.0 N<br>a <sub>ref</sub> = 0.010 |                        |     |
|--|---|------------------------|---|------------------------|---|------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |     |
| 0.002                                    | 0.154   | 151                    | -0.114  | 181                    | -7.5  | 201                    |     |
| .077                                     | .083  | 148                    | -.044   | 187                    | -3.7  | 201                    |     |
| .109                                     | .046  | 148                    | -.009   | 235                    | -.7   | 201                    |     |
| .190                                     | -.029   | 334                    | .088  | 343                    | 8.0   | 86                     |     |
| .244                                     | -.083   | 331                    | .149  | 345                    | 10.0  | 139                    |     |
| .317                                     | -.144   | 331                    | .132  | 340                    | 3.5   | 0                      |     |
| .371                                     | -.198   | 332                    | .158  | 340                    | -1.3  | 314                    |     |
| .448                                     | -.267   | 331                    | .202  | 4                      | -1.0  | 313                    |     |
| .581                                     | -.167   | 332                    | -.342   | 175                    | .1  | 127                    |     |
| .641                                     | -.037   | 313                    | -.596   | 177                    | -.1   | 332                    |     |
| .792                                     | .654  | 150                    | .193  | 351                    | -.1   | 270                    |     |
| .856                                     | 1.000   | 152                    | 1.000   | 358                    | 1.000   | 136                    |     |
| S-II engines                             |   |                        |   |                        |   |                        |     |
| 0.009 {                                  | Center  | 0.158                  | 152   | -0.518                 | 158   | -6.8                   | 202 |
|  | 1   | -.162                  | 331   | 1.140                  | 6   | -6.4                   | 199 |
|  | 2   | -----                  | ---   | -----                  | ---   | -----                  | --- |
|  | 3   | .179                   | 151   | -2.540                 | 157   | 7.1                    | 79  |
|  | 4   | .161                   | 151   | -1.456                 | 175   | 6.3                    | 19  |
| S-IVB engine                             |   |                        |   |                        |   |                        |     |
| 0.372                                    | -0.373  | 340                    | 2.482   | 156                    | -1.3  | 314                    |     |
| LM                                       |   |                        |   |                        |   |                        |     |
| 0.647                                    | -0.021  | 353                    | -0.754  | 176                    | -----   | ---                    |     |
| .696                                     | .150  | 149                    | -.711   | 178                    | 0.1   | 40                     |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-5.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 100-PERCENT PROPELLANT LOAD - Concluded

| (b) Yaw direction                        |  |                        |  |                        |   |                        |  |                        |     |
|--|--|------------------------|--|------------------------|---|------------------------|--|------------------------|-----|
| Normalized<br>model<br>station,<br>x'/L' | Frequency = 16.5 Hz<br>Force = 12.0 N<br>a <sub>ref</sub> = 0.373<br>g = 0.025 |                        | Frequency = 50.4 Hz<br>Force = 15.6 N<br>a <sub>ref</sub> = 0.373<br>g = 0.020 |                        | Frequency = 87.2 Hz<br>Force = 20.0 N<br>a <sub>ref</sub> = 0.014<br>g = ---- |                        | Frequency = 117.0 Hz<br>Force = 17.8 N<br>a <sub>ref</sub> = 0.192<br>g = ---- |                        |     |
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg |     |
|  | (a)  | (b)                    | (a)  | (b)                    | (a)   | (b)                    | (c)  | (b)                    |     |
| 0.002                                    | 0.147  | 345                    | -0.135   | 263                    | 3.071   | 360                    | 1.000  | 95                     |     |
| .077                                     | .078   | 342                    | -.056  | 258                    | 1.285   | 352                    | .333   | 98                     |     |
| .109                                     | .048   | 346                    | -.011  | 269                    | .357  | 331                    | .151   | 101                    |     |
| .190                                     | -.024  | 154                    | .090   | 80                     | -1.285  | 188                    | -.099  | 190                    |     |
| .244                                     | -.078  | 157                    | .157   | 76                     | -2.428  | 176                    | -.229  | 205                    |     |
| .317                                     | -.150  | 158                    | .202   | 76                     | -3.000  | 165                    | -.224  | 198                    |     |
| .371                                     | -.196  | 161                    | .191   | 79                     | -2.928  | 139                    | -.255  | 162                    |     |
| .448                                     | -.268  | 160                    | .180   | 105                    | -.786   | 131                    | -.104  | 151                    |     |
| .496                                     | -.252  | 161                    | .101   | 182                    | .857  | 318                    | .021   | 354                    |     |
| .581                                     | -.139  | 161                    | -.360  | 256                    | .357  | 342                    | .015   | 357                    |     |
| .641                                     | -.005  | 200                    | -.561  | 260                    | .071  | 32                     | .010   | 0                      |     |
| .696                                     | .188   | 335                    | -.528  | 265                    | -.428   | 137                    | 0  | 0                      |     |
| .763                                     | .220   | 338                    | -.146  | 269                    | -.571   | 140                    | -.005  | 174                    |     |
| .827                                     | .826   | 339                    | .596   | 83                     | .500  | 20                     | .010   | 33                     |     |
| .856                                     | 1.000  | 339                    | 1.000  | 82                     | 1.000   | 338                    | .005   | 25                     |     |
| S-II engines                             |  |                        |  |                        |   |                        |  |                        |     |
| 0.009                                    | Center   | 0.153                  | 20   | 0.292                  | 105   | -8.428                 | 176  | 0.656                  | 83  |
|  | 1  | .163                   | 341  | -1.326                 | 261   | -3.428                 | 175  | -.370                  | 277 |
|  | 2  | .166                   | 340  | 2.393                  | 43  | -2.571                 | 182  | .354                   | 279 |
|  | 3  | .161                   | 21   | .865                   | 97  | -3.357                 | 179  | .427                   | 86  |
|  | 4  | .163                   | 20   | 1.011                  | 93  | -4.286                 | 165  | .406                   | 85  |
| S-IVB engine                             |  |                        |  |                        |   |                        |  |                        |     |
| 0.372                                    | -0.418   | 165                    | -3.652   | 237                    | 2.071   | 237                    | 0.042  | 205                    |     |
| LM                                       |  |                        |  |                        |   |                        |  |                        |     |
| 0.647                                    | -0.016   | 116                    | 0.708  | 84                     | 0.071   | 330                    | -0.005   | 180                    |     |
| .696                                     | -.172  | 156                    | .685   | 86                     | .428  | 19                     | .010   | 0                      |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

<sup>c</sup>Normalized with respect to amplitude at aft end of S-II.

TABLE A-6.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 50-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>x'/L' | Frequency = 17.0 Hz<br>Force = 16.0 N<br>a <sub>ref</sub> = 0.510 |                        | Frequency = 53.0 Hz<br>Force = 16.9 N<br>a <sub>ref</sub> = 0.150 |                        | Frequency = 100.0 Hz<br>Force = 14.7 N<br>a <sub>ref</sub> = 0.013 |                        |     |
|--|---|------------------------|---|------------------------|--|------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg |     |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)  | (b)                    |     |
| 0.002                                    | 0.214   | 160                    | -0.073  | 163                    | 7.770  | 213                    |     |
| .077                                     | .129  | 158                    | -.020   | 161                    | 3.615  | 209                    |     |
| .109                                     | .084  | 156                    | -----   | ---                    | .231   | 198                    |     |
| .190                                     | .008  | 90                     | .060  | 343                    | ±13.77   | 118                    |     |
| .244                                     | -.055   | 350                    | .120  | 345                    | 9.85   | 152                    |     |
| .317                                     | -.127   | 346                    | .113  | 339                    | -4.46  | 17                     |     |
| .371                                     | -.192   | 344                    | .153  | 344                    | -3.08  | 359                    |     |
| .448                                     | -.221   | 344                    | .200  | 7                      | -9.23  | 350                    |     |
| .581                                     | -.186   | 345                    | -.327   | 179                    | ±.231  | 90                     |     |
| .641                                     | -.073   | 342                    | -.573   | 180                    | ±.154  | 90                     |     |
| .792                                     | .637  | 162                    | .193  | 358                    | .462   | 235                    |     |
| .856                                     | 1.000   | 162                    | 1.000   | 360                    | 1.000  | 235                    |     |
| S-II engines                             |   |                        |   |                        |  |                        |     |
| 0.009                                    | Center  | 0.216                  | 159   | -0.407                 | 137  | 7.08                   | 32  |
|  | 1   | .217                   | 158   | -.967                  | 165  | 6.85                   | 31  |
|  | 2   | -----                  | ---   | -----                  | ---  | -----                  | --- |
|  | 3   | .249                   | 158   | -2.053                 | 179  | 7.15                   | 31  |
|  | 4   | .222                   | 158   | -1.227                 | 156  | 6.62                   | 32  |
| S-IVB engine                             |   |                        |   |                        |  |                        |     |
| 0.372                                    | -0.386  | 354                    | -2.006  | 154                    | 1.000  | 346                    |     |
| LM                                       |   |                        |   |                        |  |                        |     |
| 0.647                                    | -0.047  | 353                    | -0.747  | 179                    | ±0.076   | 90                     |     |
| .696                                     | .127  | 157                    | -.713   | 179                    | .076   | 37                     |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-6.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 50-PERCENT PROPELLANT LOAD - Concluded

(b) Yaw direction

| Normalized<br>model<br>station,<br>x'/L' | Frequency = 17.4 Hz<br>Force = 11.6 N<br>a <sub>ref</sub> = 0.346<br>g = 0.020 |                        | Frequency = 48.2 Hz<br>Force = 11.6 N<br>a <sub>ref</sub> = 0.046<br>g = 0.012 |                        | Frequency = 97.7 Hz<br>Force = 16.0 N<br>a <sub>ref</sub> = 0.100<br>g = ---- |                        | Frequency = 118.3 Hz<br>Force = 16.0 N<br>a <sub>ref</sub> = 0.387<br>g = 0.023 |                        |     |
|--|--|------------------------|--|------------------------|---|------------------------|---|------------------------|-----|
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |     |
|  | (a)  | (b)                    | (a)  | (b)                    | (c)   | (b)                    | (c)   | (b)                    |     |
| 0.002                                    | 0.283  | 252                    | -0.174   | 177                    | 1.000   | 56                     | 1.000   | 348                    |     |
| .077                                     | .188   | 251                    | -.087  | 171                    | .47   | 54                     | .351  | 348                    |     |
| .109                                     | .150   | 250                    | -.043  | 167                    | .01   | 353                    | .147  | 347                    |     |
| .190                                     | .052   | 246                    | .043   | 14                     | -.68  | 233                    | -.220   | 161                    |     |
| .244                                     | -.017  | 90                     | .109   | 2                      | -1.18   | 226                    | -.783   | 162                    |     |
| .317                                     | -.101  | 76                     | .174   | 359                    | -1.01   | 221                    | -.519   | 150                    |     |
| .371                                     | -.168  | 81                     | .174   | 27                     | -.73  | 201                    | -.465   | 137                    |     |
| .448                                     | -.277  | 75                     | .087   | 57                     | -.20  | 191                    | -.163   | 134                    |     |
| .496                                     | -.257  | 77                     | -.130  | 190                    | .16   | 26                     | .054  | 360                    |     |
| .581                                     | -.159  | 76                     | -.391  | 191                    | .05   | 23                     | .016  | 291                    |     |
| .641                                     | -.035  | 97                     | -.565  | 190                    | .01   | 35                     | ±.008   | 270                    |     |
| .696                                     | .165   | 251                    | -.500  | 193                    | -.04  | 207                    | .002  | 244                    |     |
| .763                                     | .471   | 252                    | -.109  | 199                    | -.03  | 206                    | -.002   | 64                     |     |
| .827                                     | .824   | 253                    | .630   | 6                      | .04   | 108                    | -.008   | 113                    |     |
| .856                                     | 1.000  | 253                    | 1.000  | 8                      | .11   | 37                     | -.005   | 161                    |     |
| S-II engines                             |  |                        |  |                        |   |                        |   |                        |     |
| 0.009                                    | Center   | -0.295                 | 110  | ±0.413                 | 136   | 2.12                   | 125   | -0.628                 | 169 |
|  | 1  | .315                   | 250  | -1.239                 | 179   | -.88                   | 253   | -.367                  | 169 |
|  | 2  | .329                   | 250  | ±5.848                 | 286   | -.91                   | 237   | -.346                  | 170 |
|  | 3  | -.315                  | 110  | ±.978                  | 127   | 1.17                   | 125   | -.426                  | 167 |
|  | 4  | -.318                  | 110  | ±1.239                 | 116   | 1.19                   | 127   | -.400                  | 166 |
| S-IVB engine                             |  |                        |  |                        |   |                        |   |                        |     |
| 0.372                                    | 0.410  | 283                    | 2.022  |                        | -0.27   | 190                    | -0.096  | 220                    |     |
| LM                                       |  |                        |  |                        |   |                        |   |                        |     |
| 0.647                                    | 0.017  | 297                    | 0.696  |                        | -0.02   | 210                    | -0.005  | 156                    |     |
| .696                                     | -.144  | 70                     | .652   |                        | .02   | 90                     | .002  | 44                     |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

<sup>c</sup>Normalized with respect to amplitude at aft end of S-II.

TABLE A-7.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 0-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>x'/L' | Frequency = 17.9 Hz<br>Force = 14.2 N<br>a <sub>ref</sub> = 0.396 |                               | Frequency = 53.0 Hz<br>Force = 14.2 N<br>a <sub>ref</sub> = 0.140 |                               | Frequency = 110.9 Hz<br>Force = 14.2 N<br>a <sub>ref</sub> = 0.100 |                               |     |
|--|---|-------------------------------|---|-------------------------------|--|-------------------------------|-----|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg<br>(b) | Normalized<br>amplitude   | Phase<br>angle,<br>deg<br>(b) | Normalized<br>amplitude  | Phase<br>angle,<br>deg<br>(b) |     |
|  | (a)   | (b)                           | (a)   | (b)                           | (a)  | (b)                           |     |
| 0.002                                    | 0.409   | 148                           | -0.064  | 140                           | 1.30   | 115                           |     |
| .077                                     | .295  | 145                           | -.021   | 130                           | -.59   | 360                           |     |
| .109                                     | .230  | 144                           | -.007   | 60                            | -2.09  | 338                           |     |
| .190                                     | .116  | 139                           | .071  | 345                           | -10.30   | 100                           |     |
| .244                                     | .040  | 124                           | .121  | 344                           | -----  | ---                           |     |
| .317                                     | -.066   | 345                           | .114  | 339                           | -6.48  | 335                           |     |
| .371                                     | -.144   | 337                           | .150  | 345                           | -4.89  | 334                           |     |
| .448                                     | -.253   | 333                           | .214  | 360                           | -1.81  | 331                           |     |
| .581                                     | -.199   | 334                           | -.329   | 177                           | .52  | 212                           |     |
| .641                                     | -.081   | 343                           | -.586   | 180                           | .27  | 214                           |     |
| .792                                     | .634  | 150                           | .207  | 360                           | -.34   | 99                            |     |
| .856                                     | 1.000   | 150                           | 1.000   | 2                             | 1.00   | 207                           |     |
| S-II engines                             |   |                               |   |                               |  |                               |     |
| 0.009 {                                  | Center  | 0.429                         | 147   | -0.443                        | 128  | -0.46                         | 48  |
|  | 1   | .437                          | 147   | -.836                         | 149  | -.32                          | 68  |
|  | 2   | .444                          | 148   | -1.014                        | 146  | -.42                          | 48  |
|  | 3   | .492                          | 148   | -1.650                        | 163  | -.68                          | 30  |
|  | 4   | .442                          | 147   | -1.136                        | 140  | -.60                          | 323 |
| S-IVB engine                             |   |                               |   |                               |  |                               |     |
| 0.372                                    | -0.323  | 340                           | -2.314  | 158                           | -0.99  | 311                           |     |
| LM                                       |   |                               |   |                               |  |                               |     |
| 0.647                                    | -0.066  | 342                           | -0.757  | 180                           | 0.26   | 205                           |     |
| .696                                     | .106  | 148                           | -.714   | 178                           | .04  | 10                            |     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-7.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION II  
WITH 0-PERCENT PROPELLANT LOAD - Concluded

(b) Yaw direction

| Normalized<br>model<br>station,<br>x'/L' | Frequency = 18.1 Hz<br>Force = 12.9 N<br>a <sub>ref</sub> = 0.579<br>g = 0.020 |                        |     | Frequency = 48.2 Hz<br>Force = 11.1 N<br>a <sub>ref</sub> = 0.044<br>g = ---- |                        | Frequency = 100.3 Hz<br>Force = 11.1 N<br>a <sub>ref</sub> = 0.045<br>g = 0.020 |                        |
|--|--|------------------------|-----|---|------------------------|---|------------------------|
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg |     | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |
|  | (a)  | (b)                    |     | (a)   | (b)                    | (a)   | (b)                    |
| 0.002                                    | 0.181  | 345                    |     | -0.273  | 251                    | 1.200   | 250                    |
| .077                                     | .130   | 345                    |     | -.159   | 247                    | .067  | 204                    |
| .109                                     | .107   | 345                    |     | -.136   | 241                    | -.822   | 86                     |
| .190                                     | .052   | 339                    |     | .068  | 173                    | -3.27   | 81                     |
| .244                                     | .014   | 314                    |     | .091  | 131                    | -4.91   | 83                     |
| .317                                     | -.038  | 180                    |     | .159  | 103                    | -4.29   | 77                     |
| .371                                     | -.071  | 180                    |     | .159  | 103                    | -3.27   | 73                     |
| .448                                     | -.133  | 174                    |     | .136  | 157                    | -.889   | 80                     |
| .496                                     | -.128  | 177                    |     | -.114   | 237                    | .711  | 270                    |
| .581                                     | -.088  | 174                    |     | -.386   | 289                    | .333  | 285                    |
| .641                                     | -.026  | 185                    |     | -.545   | 295                    | .089  | 296                    |
| .696                                     | .069   | 349                    |     | -.500   | 284                    | -.120   | 109                    |
| .763                                     | .221   | 352                    |     | -.114   | 289                    | -.511   | 102                    |
| .827                                     | .406   | 350                    |     | .614  | 114                    | .356  | 286                    |
| .856                                     | 1.000  | 351                    |     | 1.000   | 102                    | 1.000   | 288                    |
| S-II engines                             |  |                        |     |   |                        |   |                        |
| 0.009 {                                  | Center   | 0.192                  | 7   | 0.636   | 129                    | -2.22   | 66                     |
|  | 1  | .207                   | 354 | -1.977  | 241                    | -.778   | 78                     |
|  | 2  | .218                   | 354 | ±6.273  | 18                     | -.911   | 68                     |
|  | 3  | .204                   | 6   | 1.477   | 117                    | -1.18   | 71                     |
|  | 4  | .207                   | 7   | 1.591   | 116                    | -1.16   | 64                     |
| S-IVB engine                             |  |                        |     |   |                        |   |                        |
| 0.372                                    | -0.193   | 183                    |     | 4.250   | 192                    | 1.04  | 300                    |
| LM                                       |  |                        |     |   |                        |   |                        |
| 0.647                                    | 0.016  | 19                     |     | 0.704   | 99                     | -0.111  | 79                     |
| .696                                     | -.060  | 168                    |     | .659  | 103                    | -.067   | 56                     |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-8.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION III  
WITH 100-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>$x''/L''$ | Frequency = 38.0 Hz<br>Force = 9.3 N<br>$a_{ref} = 0.319$ |                        | Frequency = 81.2 Hz<br>Force = 8.0 N<br>$a_{ref} = 0.310$ |                        | Frequency = 99.0 Hz<br>Force = 5.8 N<br>$a_{ref} = 0.885$ |                        |
|--|---|------------------------|---|------------------------|---|------------------------|
|  | Normalized<br>amplitude                                   | Phase<br>angle,<br>deg | Normalized<br>amplitude                                   | Phase<br>angle,<br>deg | Normalized<br>amplitude                                   | Phase<br>angle,<br>deg |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |
| 0.017  | 0.460   | 93                     | -2.74   | 68                     | -0.434  | 180                    |
| .172   | .100  | 112                    | -.203   | 84                     | -.052   | 207                    |
| .271   | -.272   | 267                    | .754  | 250                    | .091  | 348                    |
| .361   | -.106   | 251                    | .912  | 254                    | .111  | 9                      |
| .444   | -.010   | 292                    | .787  | 255                    | .080  | 24                     |
| .565   | -.498   | 272                    | .048  | 274                    | -.080   | 127                    |
| .676   | -.382   | 272                    | -.355   | 74                     | -.277   | 154                    |
| .812   | .038  | 94                     | -.364   | 78                     | -.472   | 158                    |
| .872   | .010  | 113                    | .068  | 225                    | -.151   | 134                    |
| .942   | .658  | 93                     | .510  | 250                    | .418  | 353                    |
| 1.000  | 1.000   | 94                     | 1.000   | 256                    | 1.000   | 343                    |
| S-IVB engine                                 |   |                        |   |                        |   |                        |
| 0.017  | ±1.777  | 174                    | 0.516   | 292                    | 0.063   | 47                     |
| LM   |   |                        |   |                        |   |                        |
| 0.576  | -0.570  | 275                    | -0.639  | 79                     | -0.064  | 141                    |
| .676   | -.436   | 275                    | -1.06   | 116                    | -.139   | 166                    |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-8.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION III  
WITH 100-PERCENT PROPELLANT LOAD - Concluded

(b) Yaw direction

| Normalized<br>model<br>station,<br>$x''/L''$ | Frequency = 40.0 Hz<br>Force = 13.4 N<br>$a_{ref} = 0.465$<br>$g = 0.025$ |                        | Frequency = 76.3 Hz<br>Force = 17.6 N<br>$a_{ref} = 0.629$<br>$g = 0.034$ |                        | Frequency = 114.7 Hz<br>Force = 13.3 N<br>$a_{ref} = 0.535$<br>$g = ----$ |                        |
|--|---|------------------------|---|------------------------|---|------------------------|
|  | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)   | (b)                    |
| 0.017  | 0.47  | 108                    | -0.591  | 101                    | -1.18   | 226                    |
| .172   | .11   | 127                    | -.068   | 127                    | -.110   | 246                    |
| .271   | -.09  | 275                    | .432  | 289                    | .308  | 132                    |
| .361   | -.27  | 285                    | .461  | 286                    | .515  | 108                    |
| .444   | -.38  | 290                    | .279  | 291                    | -----   | ---                    |
| .565   | -.49  | 290                    | -.185   | 104                    | .295  | 97                     |
| .676   | -.37  | 293                    | -.448   | 111                    | .060  | 114                    |
| .812   | .02   | 99                     | -.341   | 110                    | -.204   | 277                    |
| .872   | .34   | 113                    | .073  | 276                    | -.069   | 318                    |
| .942   | .65   | 110                    | .548  | 235                    | .257  | 81                     |
| 1.000  | 1.000   | 112                    | 1.00  | 239                    | 1.00  | 95                     |
| S-IVB engine                                 |   |                        |   |                        |   |                        |
| 0.017  | -----   | ---                    | -0.205  | 62                     | 0.278   | 90                     |
| LM   |   |                        |   |                        |   |                        |
| 0.576  | -0.56   | 292                    | -0.78   | 108                    | 0.088   | 107                    |
| .676   | -.47  | 294                    | -.89  | 111                    | -.071   | 249                    |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-9.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION III  
WITH 50-PERCENT PROPELLANT LOAD

| (a) Pitch direction                          |   |                               |   |                               |
|--|---|-------------------------------|---|-------------------------------|
| Normalized<br>model<br>station,<br>$x''/L''$ | Frequency = 40.4 Hz<br>Force = 7.5 N<br>$a_{ref} = 0.574$ |                               | Frequency = 98.4 Hz<br>Force = 6.6 N<br>$a_{ref} = 1.160$ |                               |
|  | Normalized<br>amplitude<br>(a)                            | Phase<br>angle,<br>deg<br>(b) | Normalized<br>amplitude<br>(a)                            | Phase<br>angle,<br>deg<br>(b) |
| 0.017  | 0.554   | 90                            | -0.263  | 134                           |
| .172   | .153  | 105                           | .075  | 239                           |
| .271   | -.265   | 254                           | .211  | 293                           |
| .361   | -.089   | 225                           | .203  | 284                           |
| .444   | -.423   | 258                           | .156  | 306                           |
| .565   | -.538   | 261                           | -.076   | 33                            |
| .676   | -.451   | 261                           | -.272   | 78                            |
| .812   | -.012   | 262                           | -.471   | 86                            |
| .872   | .331  | 82                            | -.166   | 91                            |
| .942   | .641  | 82                            | .431  | 267                           |
| 1.000  | 1.000   | 83                            | 1.000   | 267                           |
| S-IVB engine                                 |   |                               |   |                               |
| 0.017  | -0.233  | 221                           | -0.147  | 49                            |
| LM   |   |                               |   |                               |
| 0.576  | -0.632  | 262                           | -0.058  | 52                            |
| .676   | -.522   | 305                           | -.149   | 91                            |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-9.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION III  
WITH 50-PERCENT PROPELLANT LOAD - Concluded

| (b) Yaw direction                        |   |                               |   |                               |  |                               |
|--|---|-------------------------------|---|-------------------------------|--|-------------------------------|
| Normalized<br>model<br>station,<br>x"/L" | Frequency = 43.5 Hz<br>Force = 5.8 N<br>a <sub>ref</sub> = 0.460<br>g = 0.025 |                               | Frequency = 86.1 Hz<br>Force = 12.0 N<br>a <sub>ref</sub> = 0.430<br>g = ---- |                               | Frequency = 91.2 Hz<br>Force = 10.7 N<br>a <sub>ref</sub> = 0.428<br>g = 0.035 |                               |
|  | Normalized<br>amplitude<br>(a)  | Phase<br>angle,<br>deg<br>(b) | Normalized<br>amplitude<br>(a)  | Phase<br>angle,<br>deg<br>(b) | Normalized<br>amplitude<br>(a)   | Phase<br>angle,<br>deg<br>(b) |
| 0.017                                    | 0.56  | 70                            | -0.91   | 130                           | -2.34  | 161                           |
| .172                                     | .15   | 78                            | .45   | 282                           | .22  | 289                           |
| .271                                     | -.08  | 245                           | 1.07  | 306                           | 1.52   | 348                           |
| .361                                     | -.26  | 249                           | 1.25  | 300                           | 1.92   | 340                           |
| .444                                     | ---   | ---                           | ----  | ---                           | ----   | ---                           |
| .565                                     | -.52  | 252                           | .13   | 330                           | .13  | 34                            |
| .676                                     | -.42  | 255                           | -.31  | 108                           | -.69   | 152                           |
| .812                                     | -.04  | 264                           | -.37  | 104                           | -1.16  | 151                           |
| .872                                     | .30   | 75                            | .17   | 11                            | -1.31  | 155                           |
| .942                                     | .64   | 72                            | .58   | 313                           | -.53   | 136                           |
| 1.000                                    | 1.00  | 74                            | 1.00  | 302                           | 1.00   | 342                           |
| S-IVB engine                             |   |                               |   |                               |  |                               |
| 0.017                                    | ---   | ---                           | ----  | ---                           | ----   | ---                           |
| LM                                       |   |                               |   |                               |  |                               |
| 0.576                                    | 0.61  | 106                           | -0.55   | 121                           | 0.60   |                               |
| .676                                     | -.54  | 256                           | -.79  | 125                           | .86  |                               |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-10.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION III  
WITH 0-PERCENT PROPELLANT LOAD

(a) Pitch direction

| Normalized<br>model<br>station,<br>$x''/L''$ | Frequency = 41.2 Hz<br>Force = 5.1 N<br>$a_{ref} = 0.841$ |                        | Frequency = 96.1 Hz<br>Force = 6.0 N<br>$a_{ref} = 0.686$ |                        | Frequency = 103.3 Hz<br>Force = 6.8 N<br>$a_{ref} = 1.046$ |                        |
|--|---|------------------------|---|------------------------|--|------------------------|
|  | Normalized<br>amplitude                                   | Phase<br>angle,<br>deg | Normalized<br>amplitude                                   | Phase<br>angle,<br>deg | Normalized<br>amplitude                                    | Phase<br>angle,<br>deg |
|  | (a)   | (b)                    | (a)   | (b)                    | (a)  | (b)                    |
| 0.017  | 0.511   | 106                    | 0.183   | 150                    | -0.245   | 118                    |
| .172   | .116  | 115                    | -.192   | 250                    | .310   | 95                     |
| .271   | -.100   | 210                    | -.421   | 279                    | .834   | 230                    |
| .361   | -.302   | 279                    | -.598   | 286                    | 1.183  | 237                    |
| .444   | -.448   | 281                    | -.392   | 284                    | .608   | 246                    |
| .565   | -.562   | 282                    | -.271   | 302                    | .204   | 233                    |
| .676   | -.469   | 282                    | -.380   | 320                    | -.132  | 29                     |
| .812   | -.230   | 290                    | -.481   | 328                    | -.460  | 51                     |
| .872   | .305  | 102                    | -.166   | 320                    | -.230  | 61                     |
| .942   | .041  | 102                    | .427  | 147                    | .459   | 97                     |
| 1.000  | 1.000   | 103                    | 1.000   | 140                    | 1.000  | 209                    |
| S-IVB engine                                 |   |                        |   |                        |  |                        |
| 0.017  | -0.861  | 245                    | $\pm 0.249$   | 26                     | 0.035  | 227                    |
| LM   |   |                        |   |                        |  |                        |
| 0.576  | -0.650  | 286                    | -0.139  | 288                    | 0.157  | ---                    |
| .676   | -.542   | 279                    | -.088   | 257                    | -.152  | ---                    |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

TABLE A-10.- NORMALIZED RESONANT RESPONSE OF CONFIGURATION III  
WITH 0-PERCENT PROPELLANT LOAD - Concluded

| (b) Yaw direction                        |  |                        |   |                        |
|--|--|------------------------|---|------------------------|
| Normalized<br>model<br>station,<br>x"/L" | Frequency = 43.5 Hz<br>Force = 6.7 N<br>a <sub>ref</sub> = 1.06<br>g = 0.025 |                        | Frequency = 99.8 Hz<br>Force = 8.0 N<br>a <sub>ref</sub> = 0.690<br>g = 0.021 |                        |
|  | Normalized<br>amplitude  | Phase<br>angle,<br>deg | Normalized<br>amplitude   | Phase<br>angle,<br>deg |
|  | (a)  | (b)                    | (a)   | (b)                    |
| 0.017                                    | 0.48   | 140                    | -0.34   | 227                    |
| .172                                     | .01  | 132                    | ±.11  | 60                     |
| .271                                     | -.09   | 304                    | .72   | 287                    |
| .361                                     | -.27   | 309                    | .87   | 284                    |
| .444                                     | -.38   | 315                    | .38   | 277                    |
| .565                                     | -.53   | 313                    | .16   | 265                    |
| .676                                     | -.42   | 316                    | -.24  | 167                    |
| .812                                     | -.05   | 321                    | -.48  | 161                    |
| .872                                     | .28  | 137                    | .31   | 214                    |
| .942                                     | .63  | 134                    | .51   | 297                    |
| 1.000                                    | 1.00   | 136                    | 1.00  | 325                    |
| S-IVB engine                             |  |                        |   |                        |
| 0.017                                    | -----  | ---                    | -----   | ---                    |
| LM                                       |  |                        |   |                        |
| 0.576                                    | -0.016   | 290                    | 0.025   | 331                    |
| .676                                     | .034   | 50                     | -.002   | 140                    |

<sup>a</sup>Normalized with respect to amplitude at forward end of CM.

<sup>b</sup>Phase angle of acceleration relative to input force.

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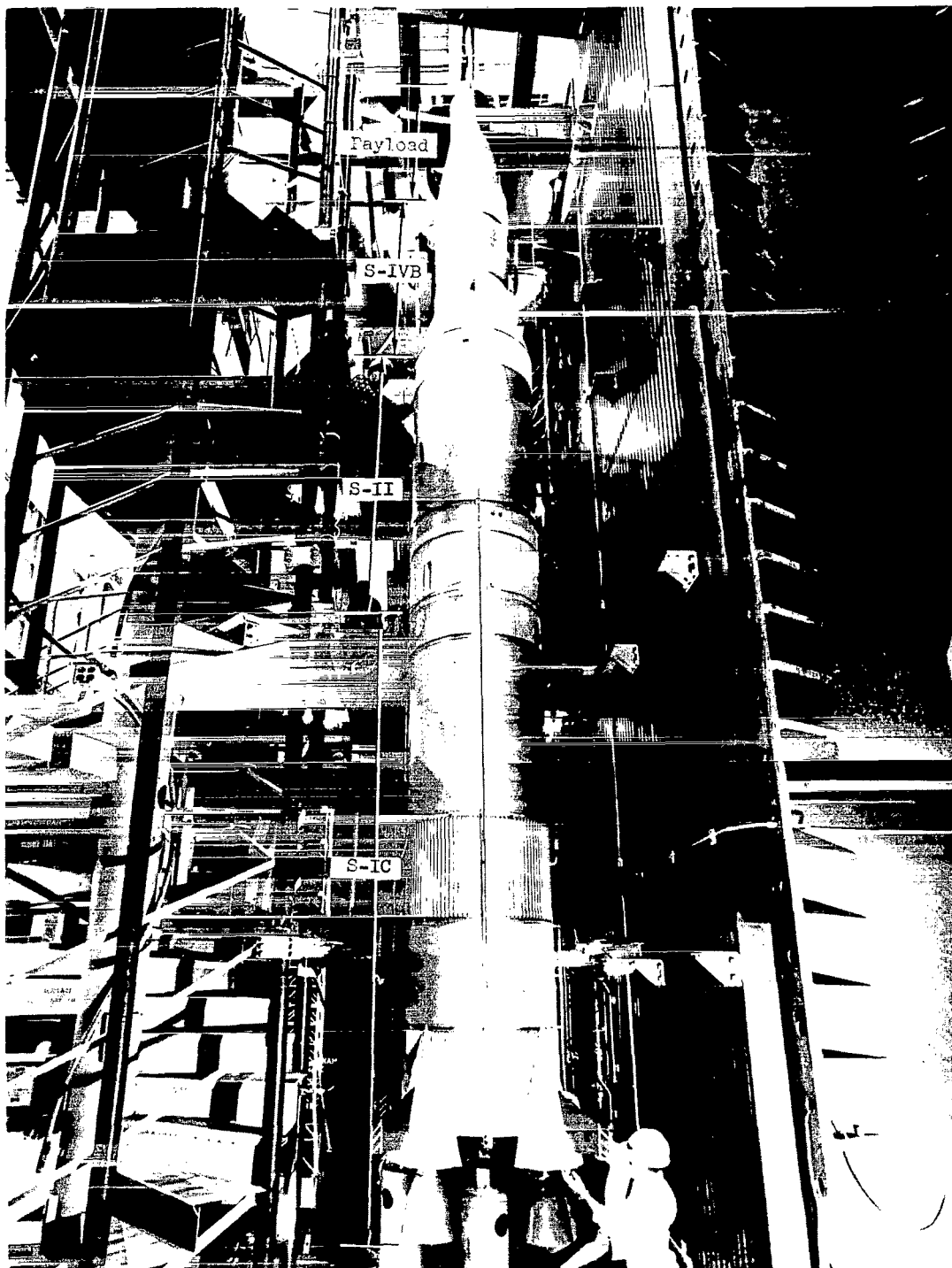


Figure 1.- 1/10-scale model of Apollo/Saturn V launch vehicle.

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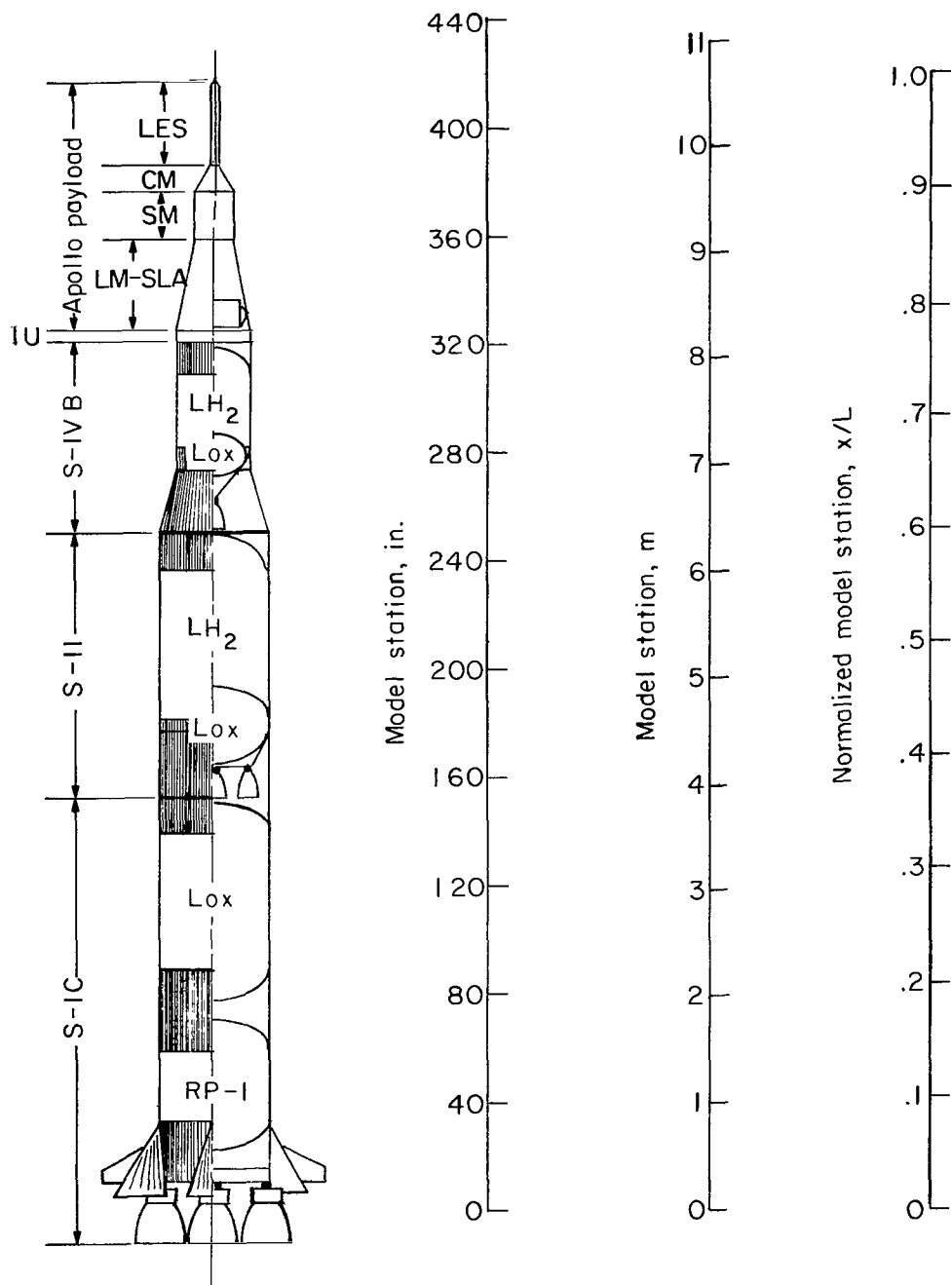


Figure 2.- Schematic of 1/10-scale Apollo/Saturn V model.

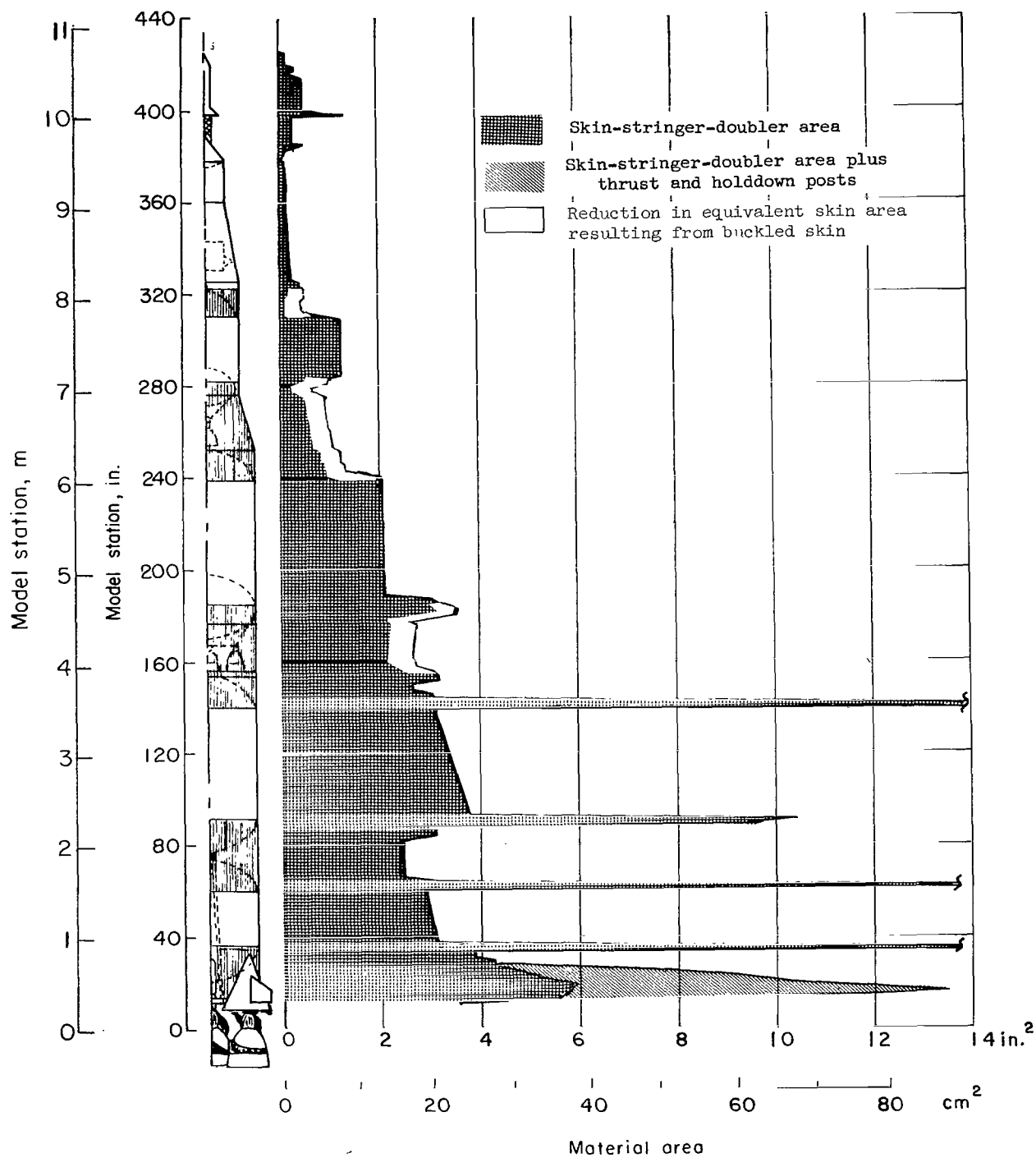
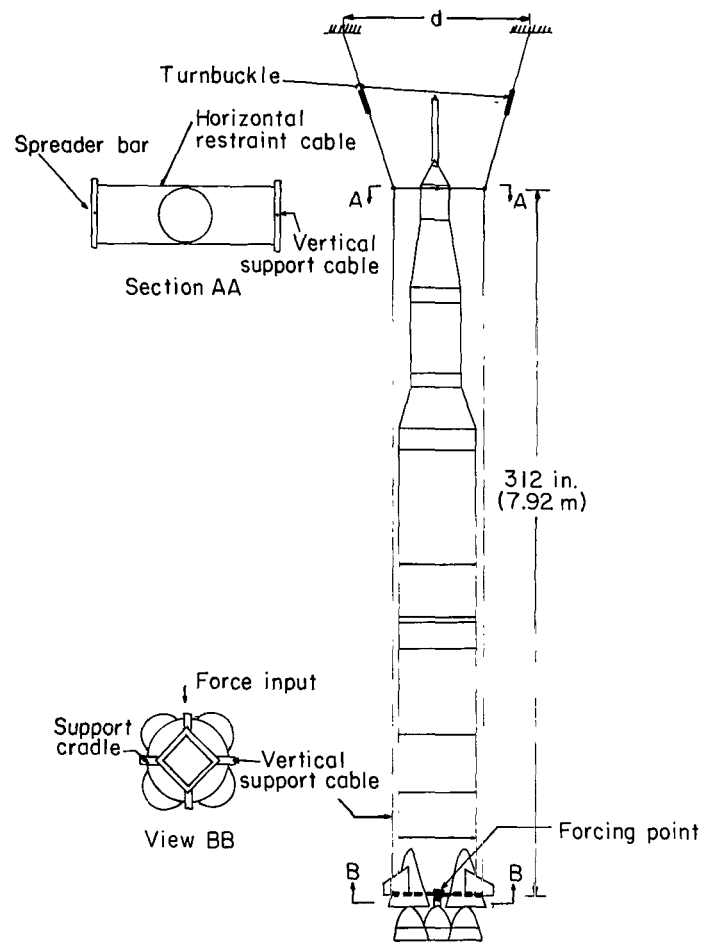
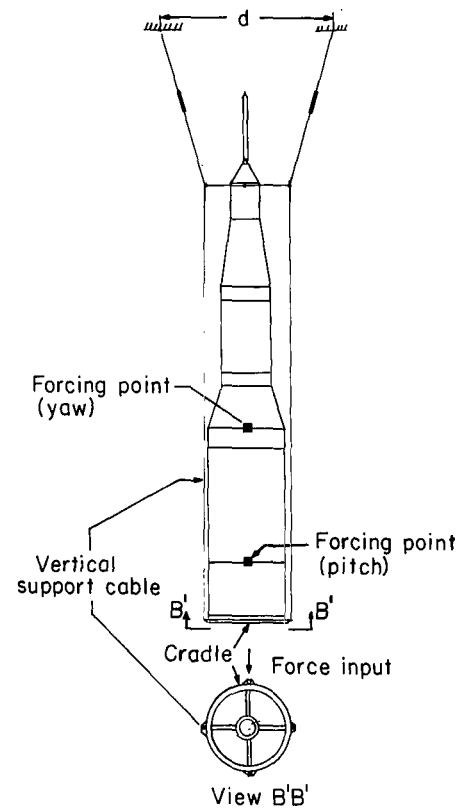


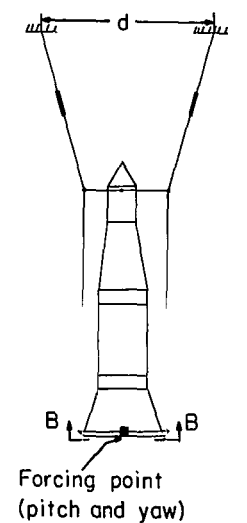
Figure 3.- Cross-sectional area of main structure.



(a) Configuration I.

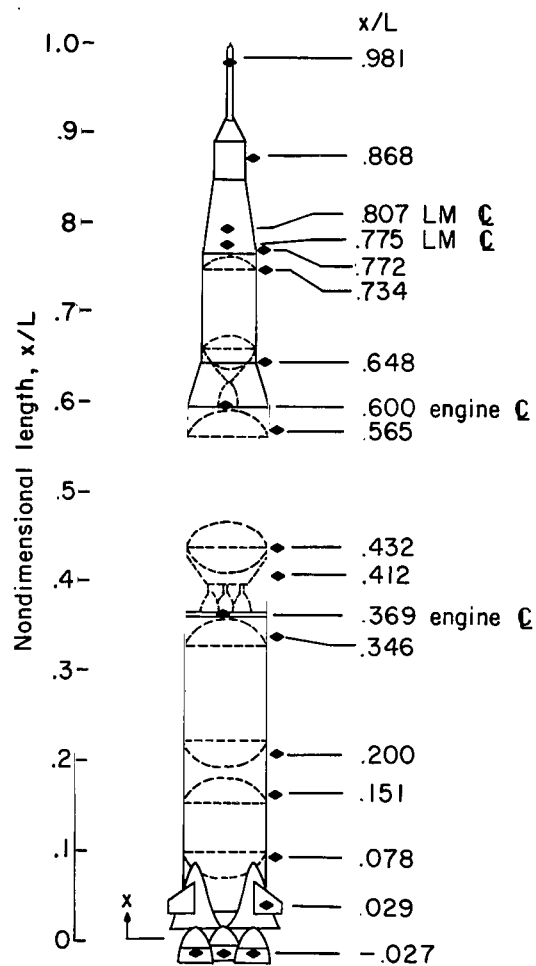


(b) Configuration II.

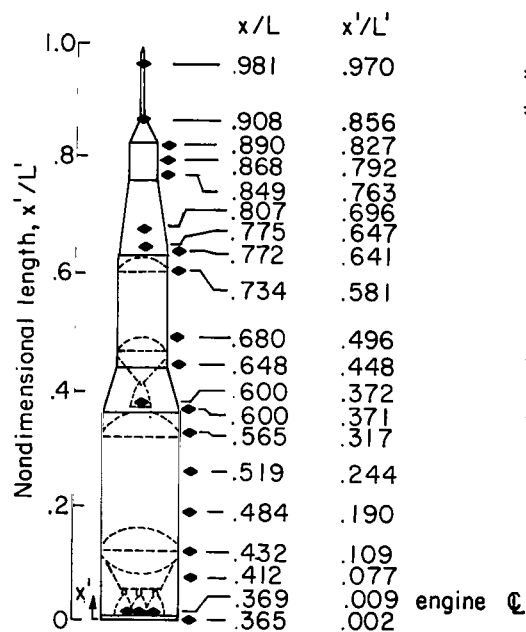


(c) Configuration III.

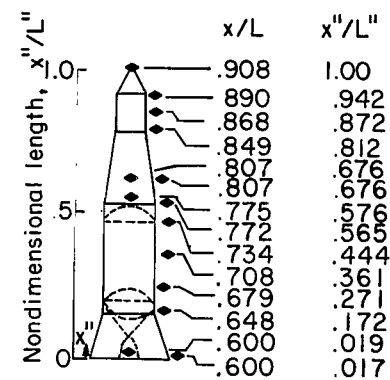
Figure 4- Suspension system.



(a) Configuration I.



(b) Configuration II.



(c) Configuration III.

Figure 5.- Locations of primary transducers.

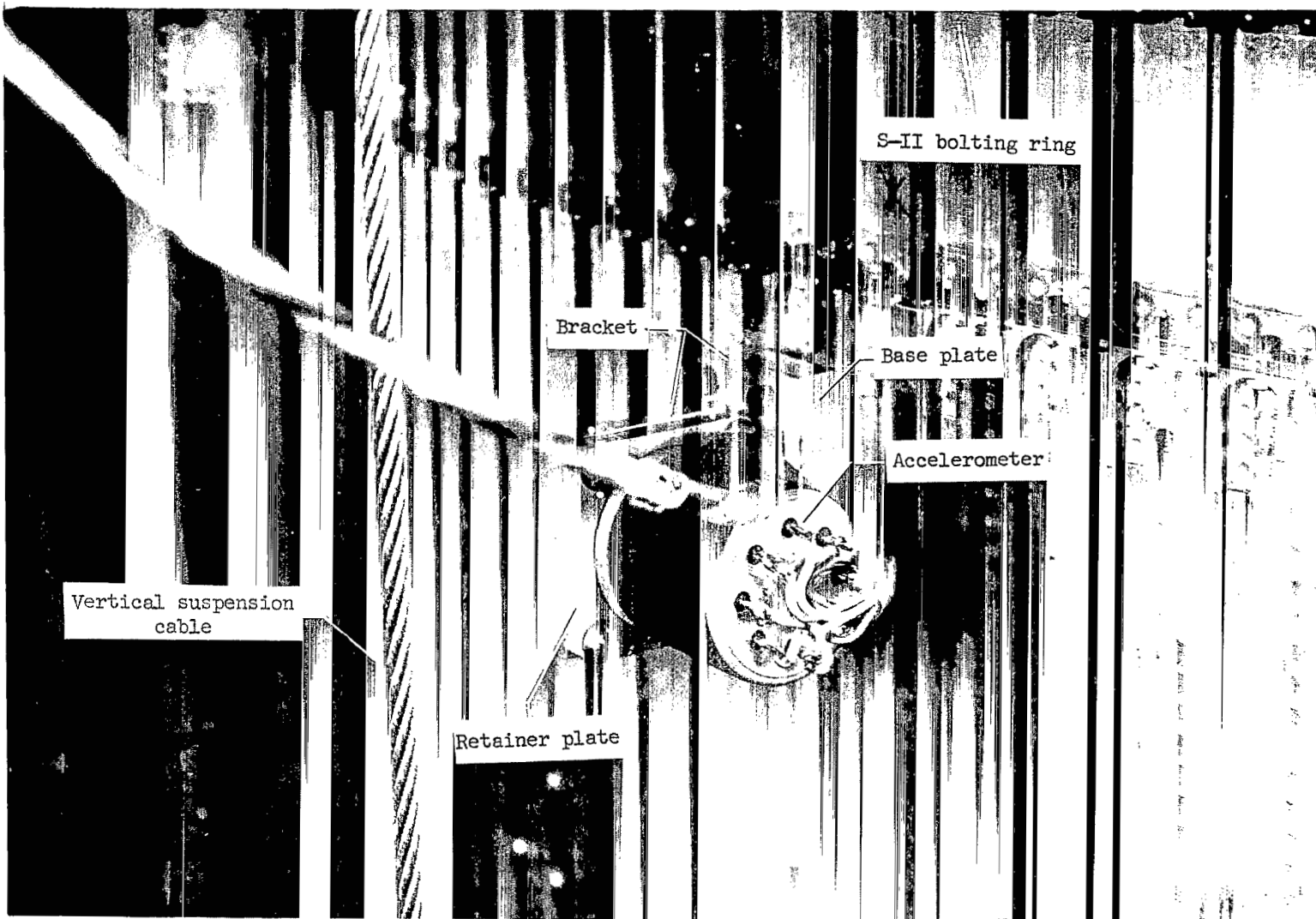


Figure 6.- Typical transducer installation.

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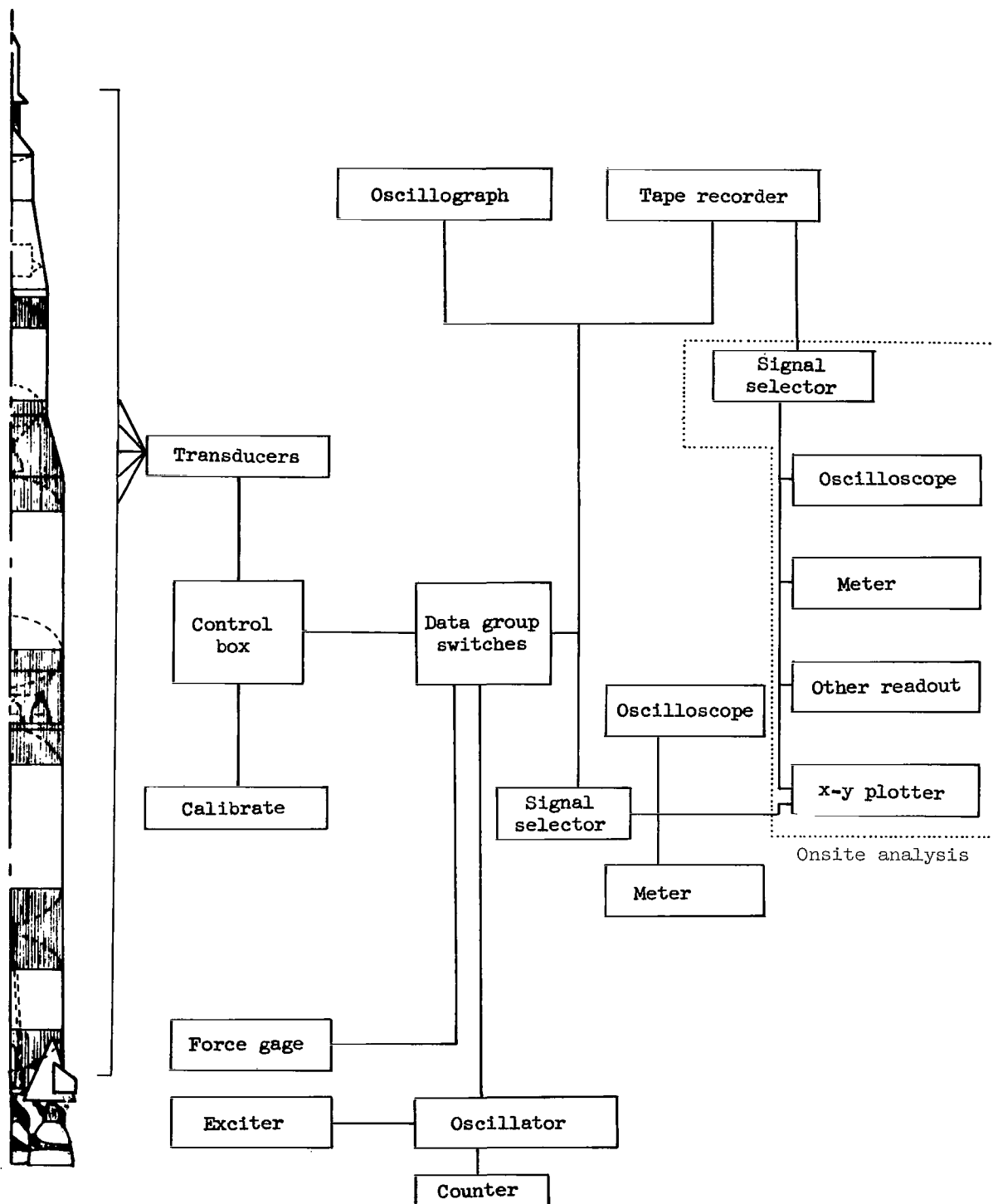


Figure 7.- Schematic diagram of instrumentation.

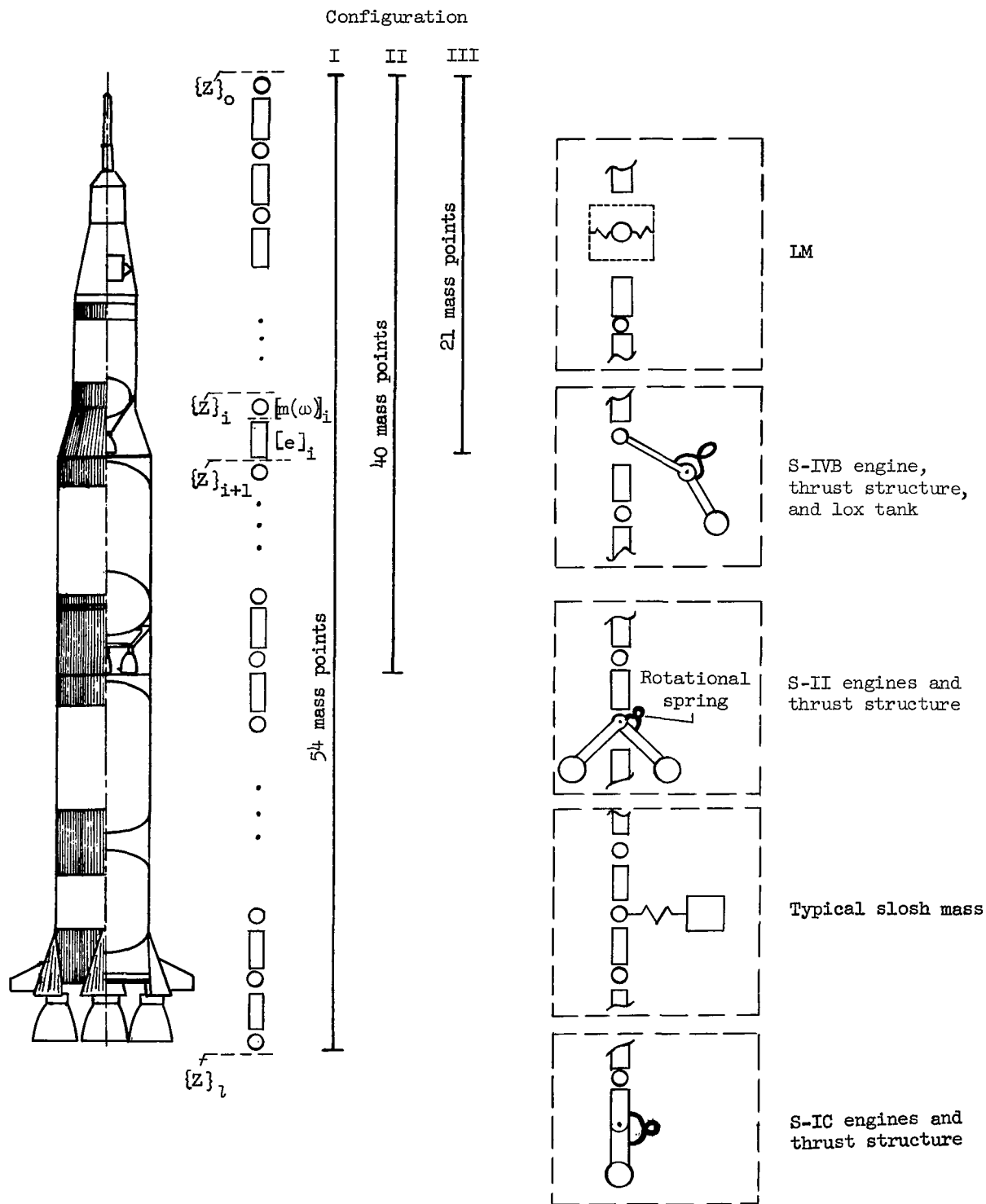


Figure 8.- Mathematical representation.

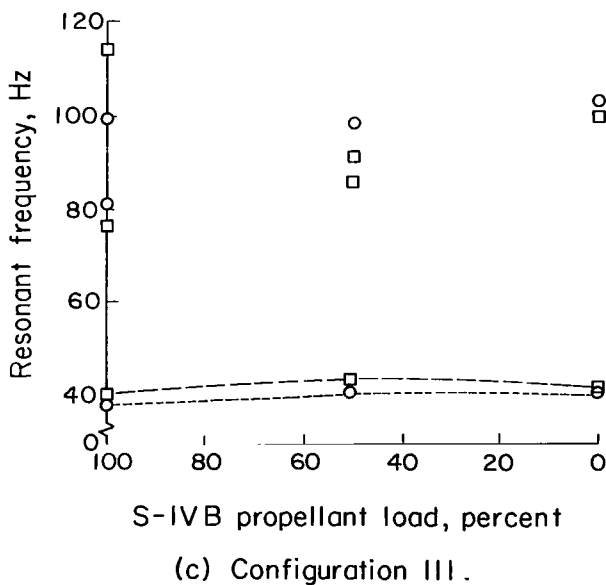
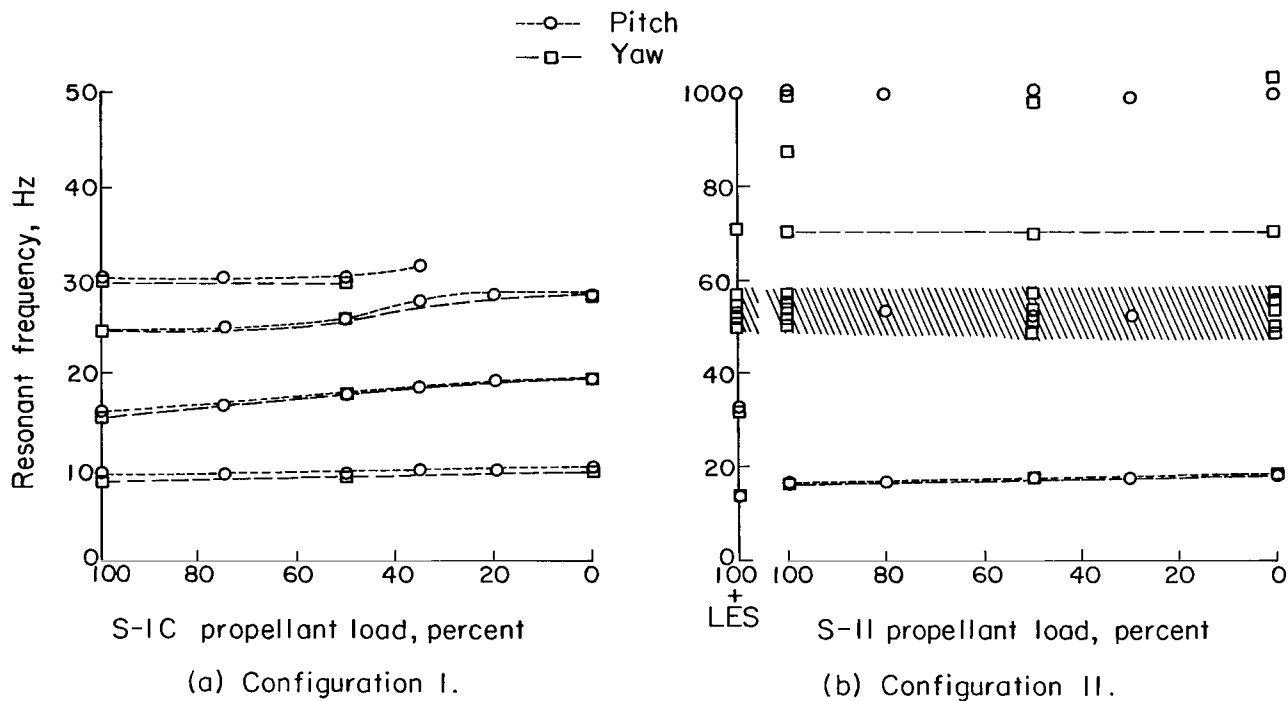


Figure 9.- Summary of measured resonant frequencies as a function of simulated propellant loading.

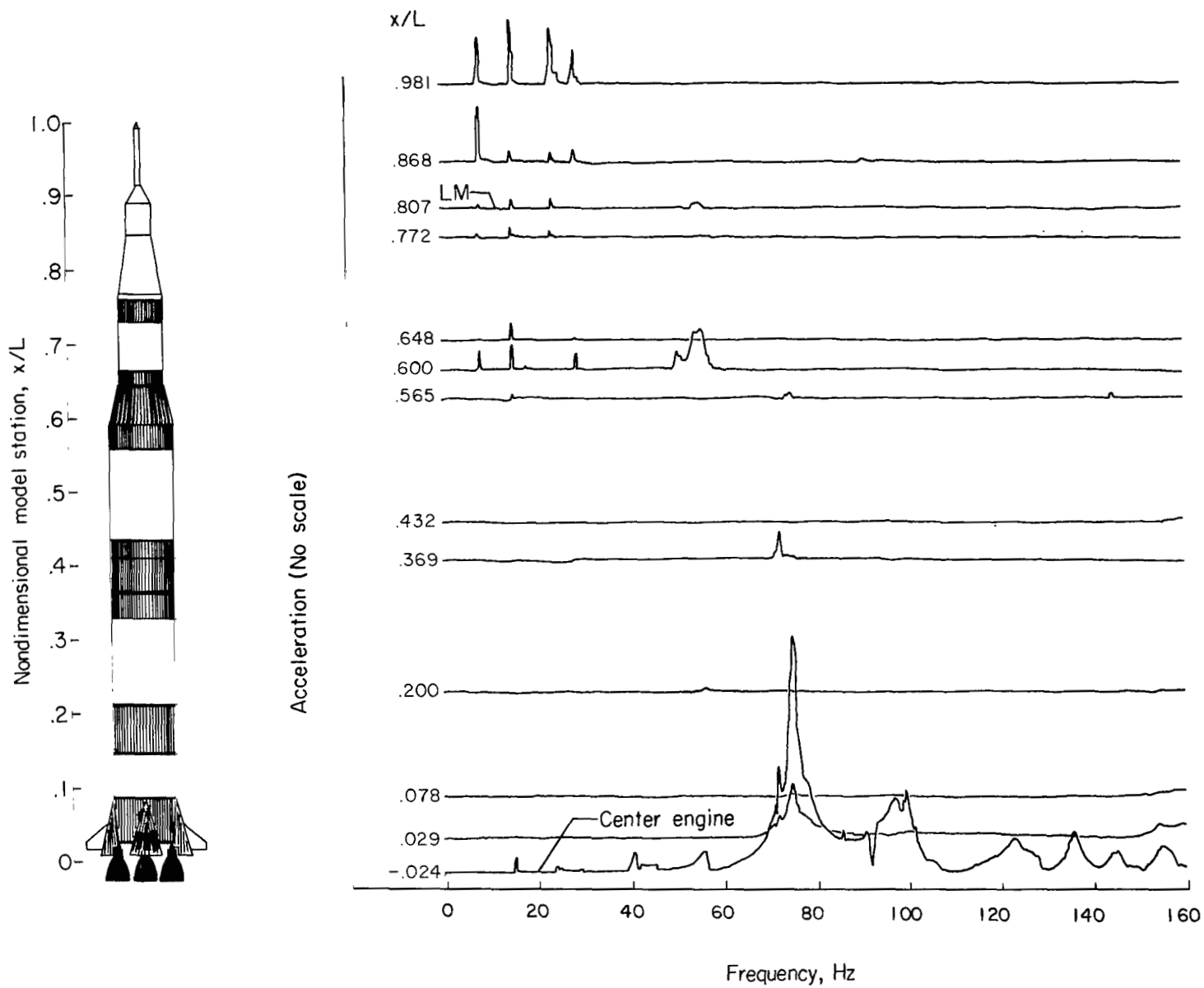


Figure 10.- Accelerations of configuration I with 100 percent propellant as a function of frequency for excitation in the yaw direction.

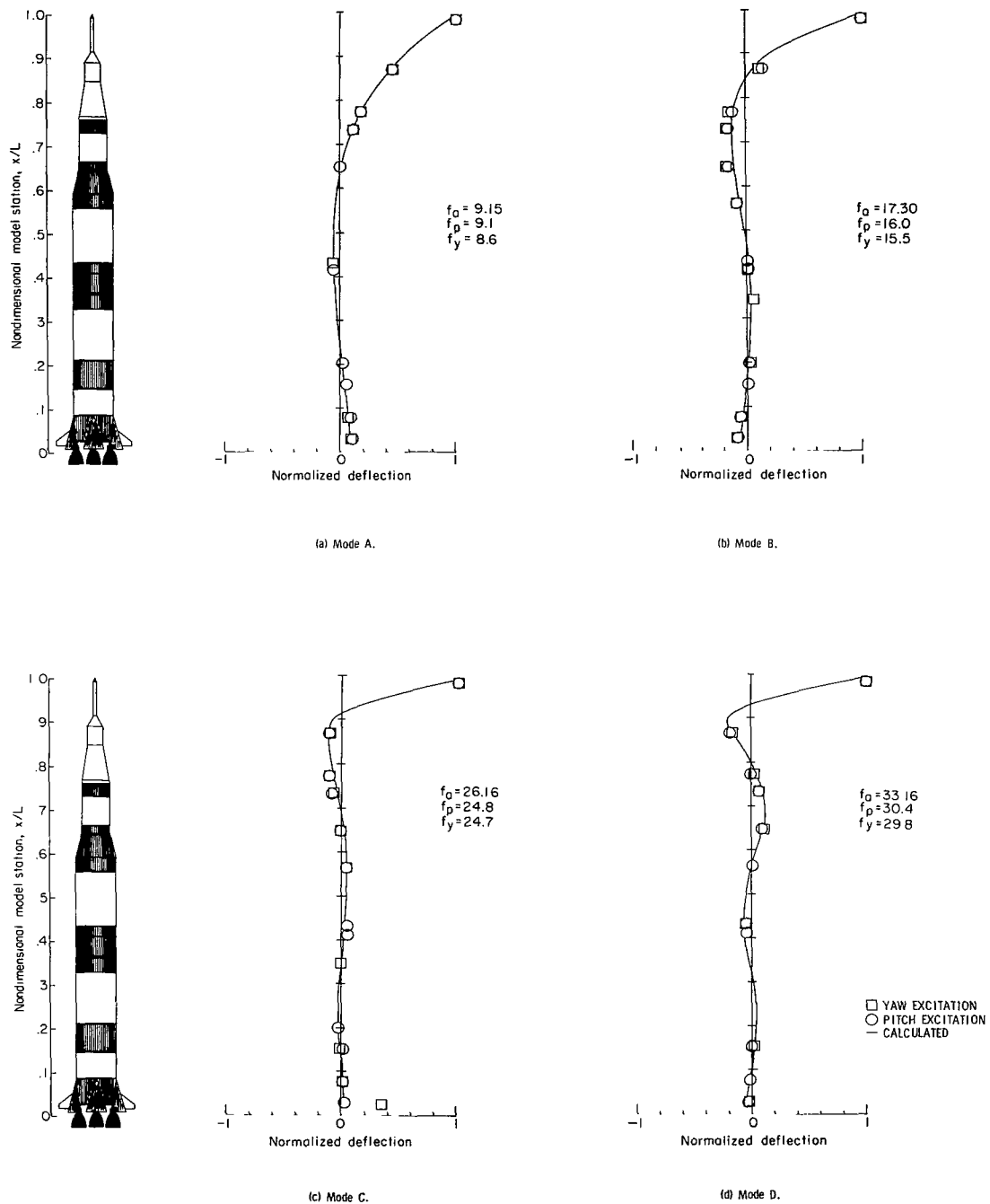


Figure 11.- Measured and calculated deflection shapes for configuration I with 100 percent propellant.

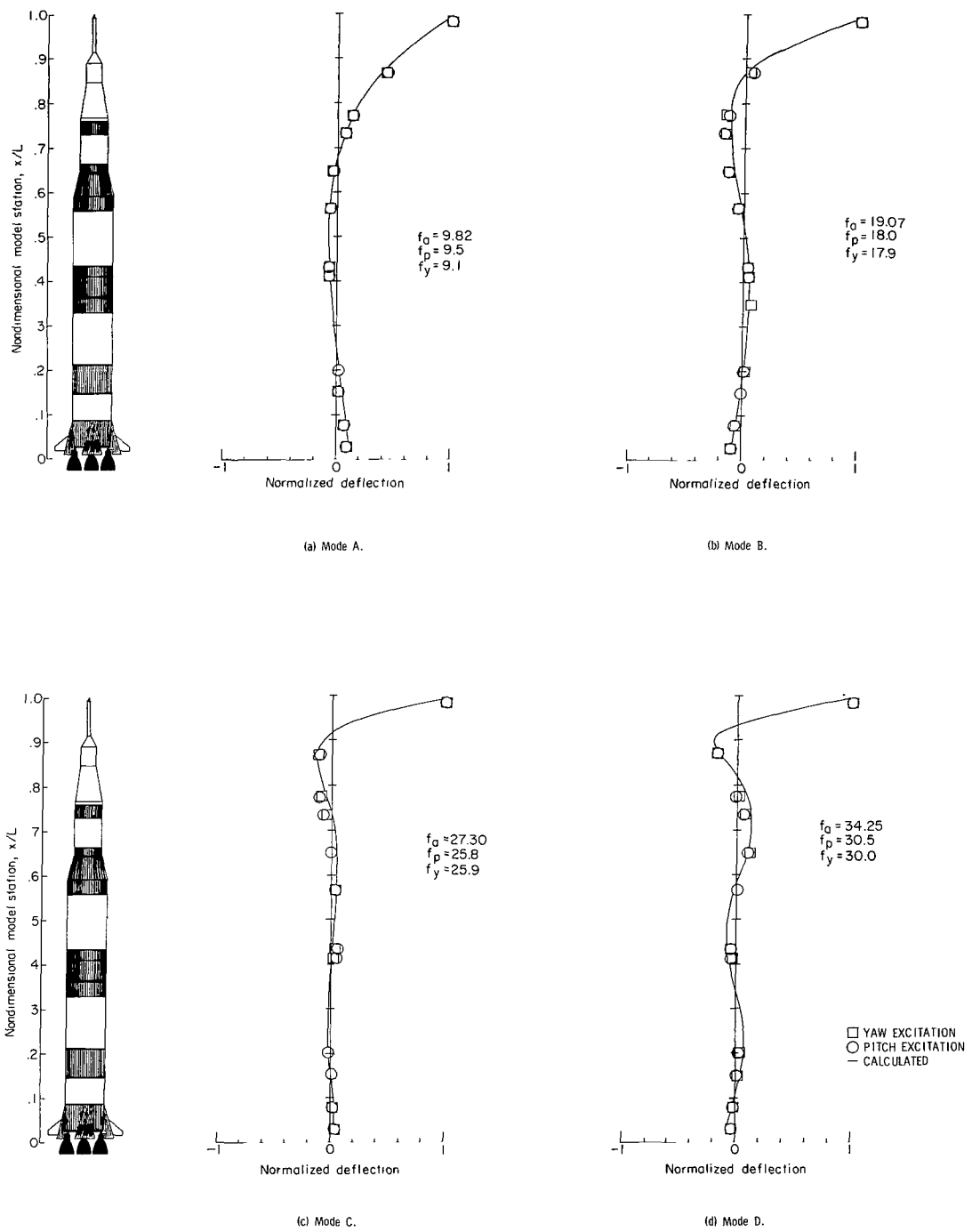


Figure 12.- Measured and calculated deflection shapes for configuration I with 50 percent propellant.

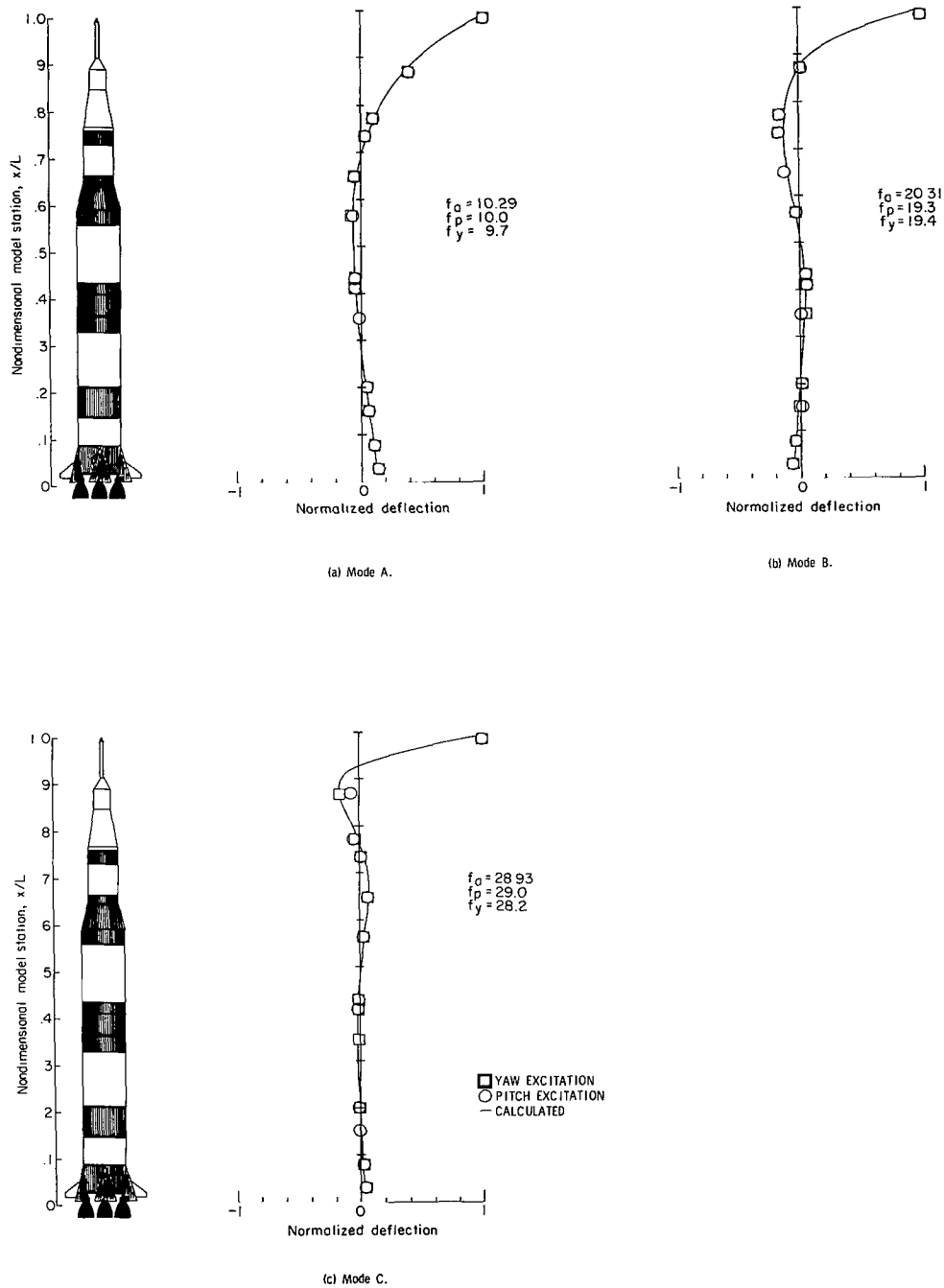


Figure 13.- Measured and calculated deflection shapes for configuration I with 0 percent propellant.

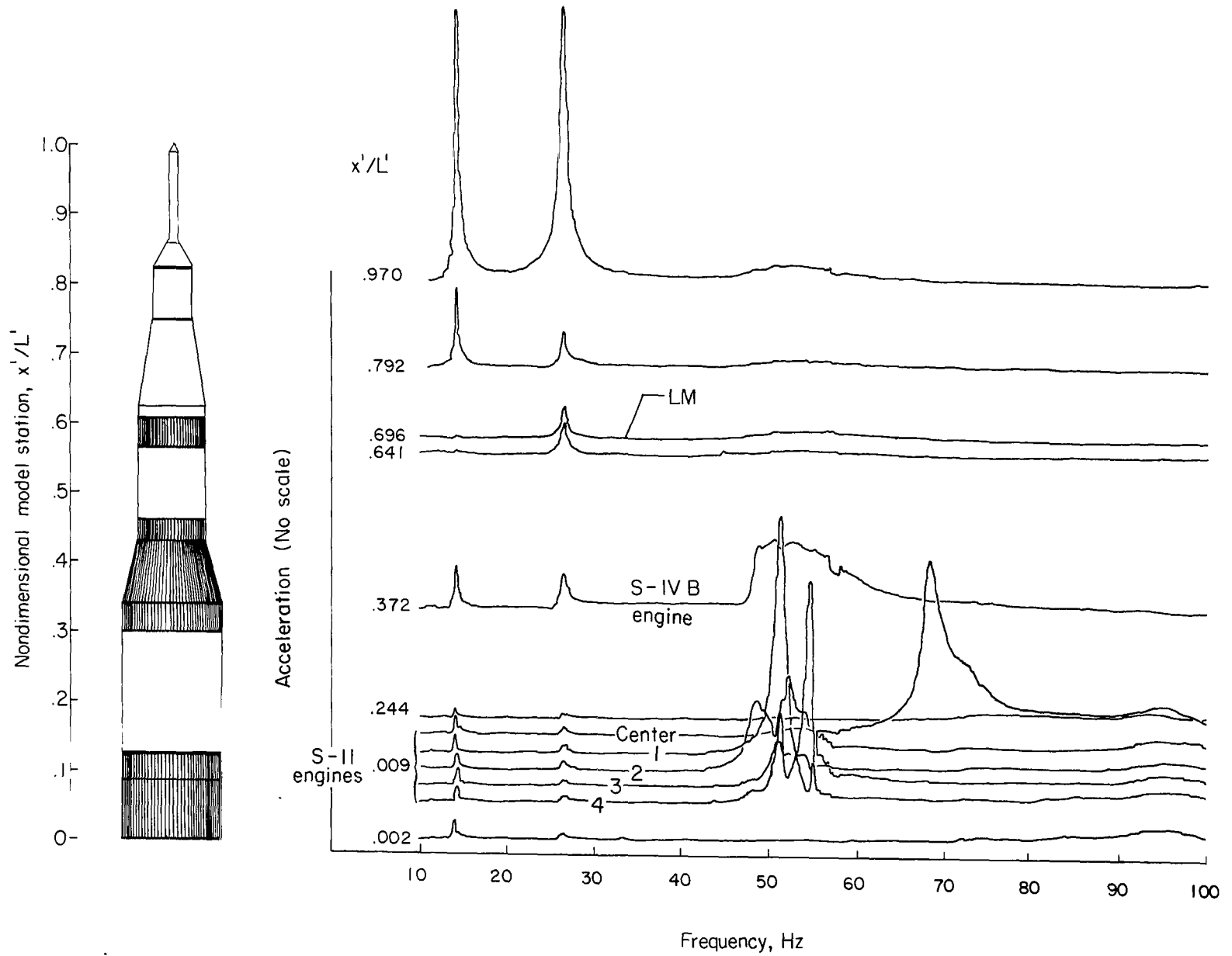


Figure 14.- Accelerations of configuration II with 100 percent propellant plus LES as a function of frequency in the yaw direction.

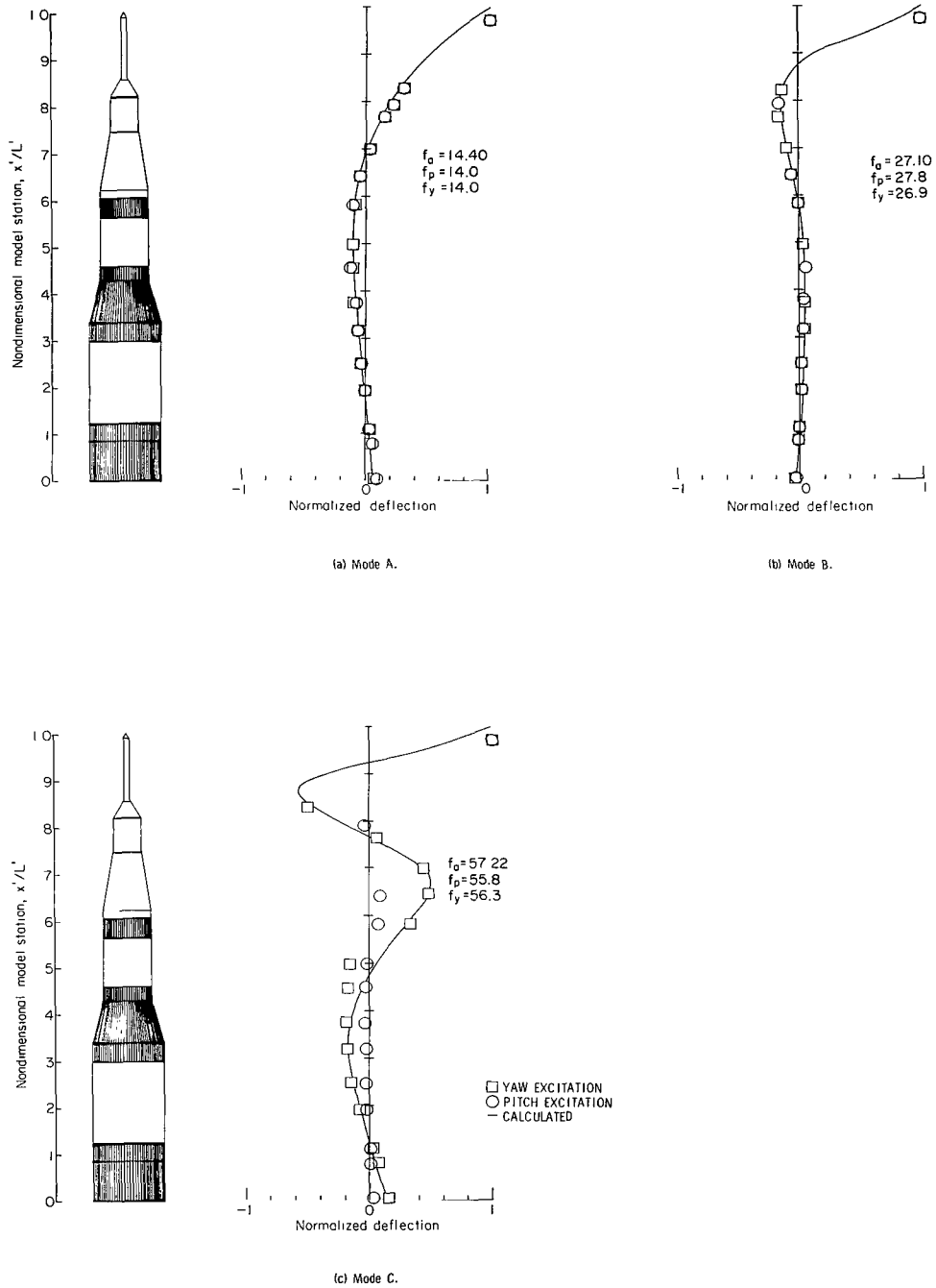


Figure 15.- Measured and calculated deflection shapes for configuration II with 100 percent propellant plus LES.

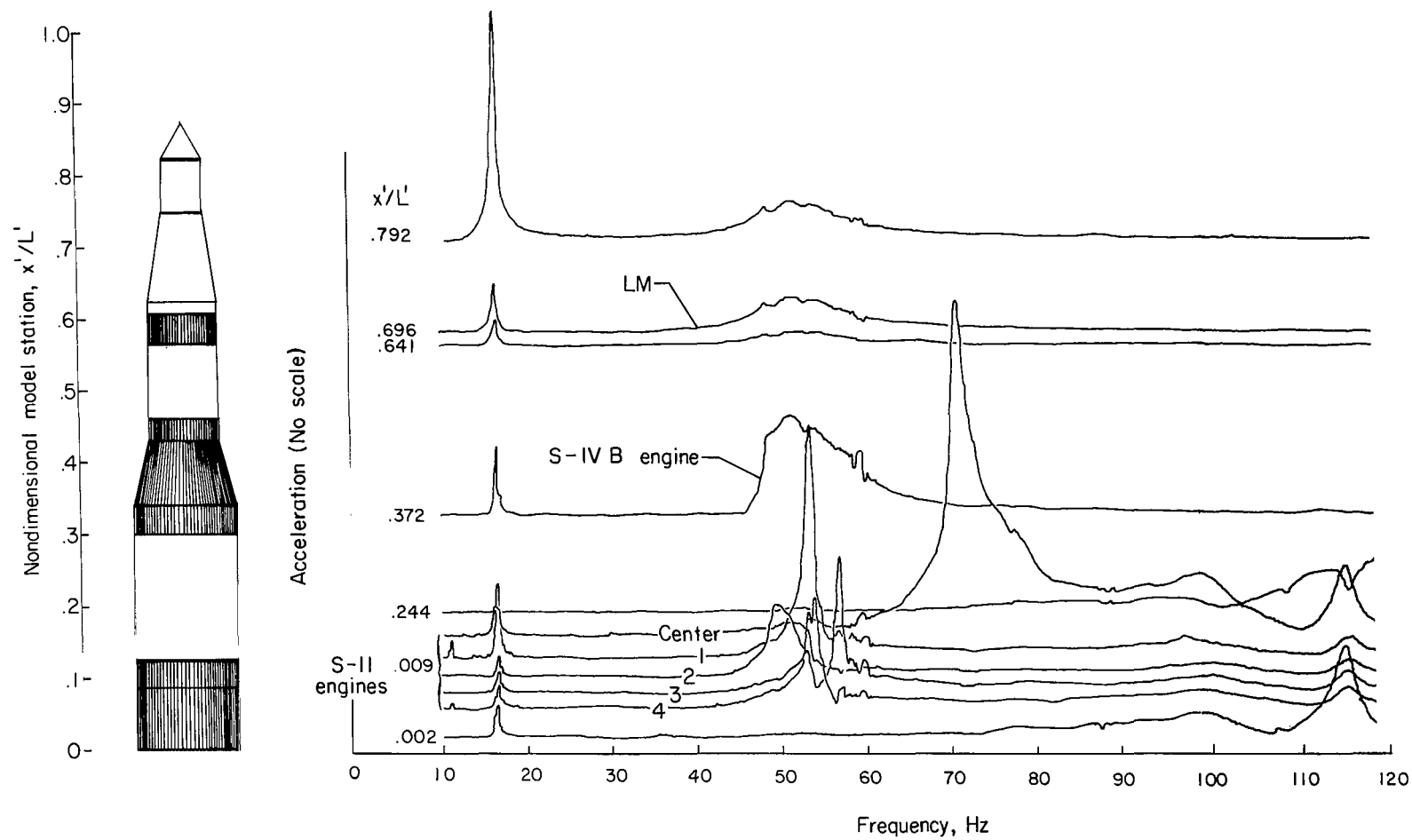


Figure 16.- Accelerations of configuration II with 100 percent propellant as a function of frequency in the yaw direction.

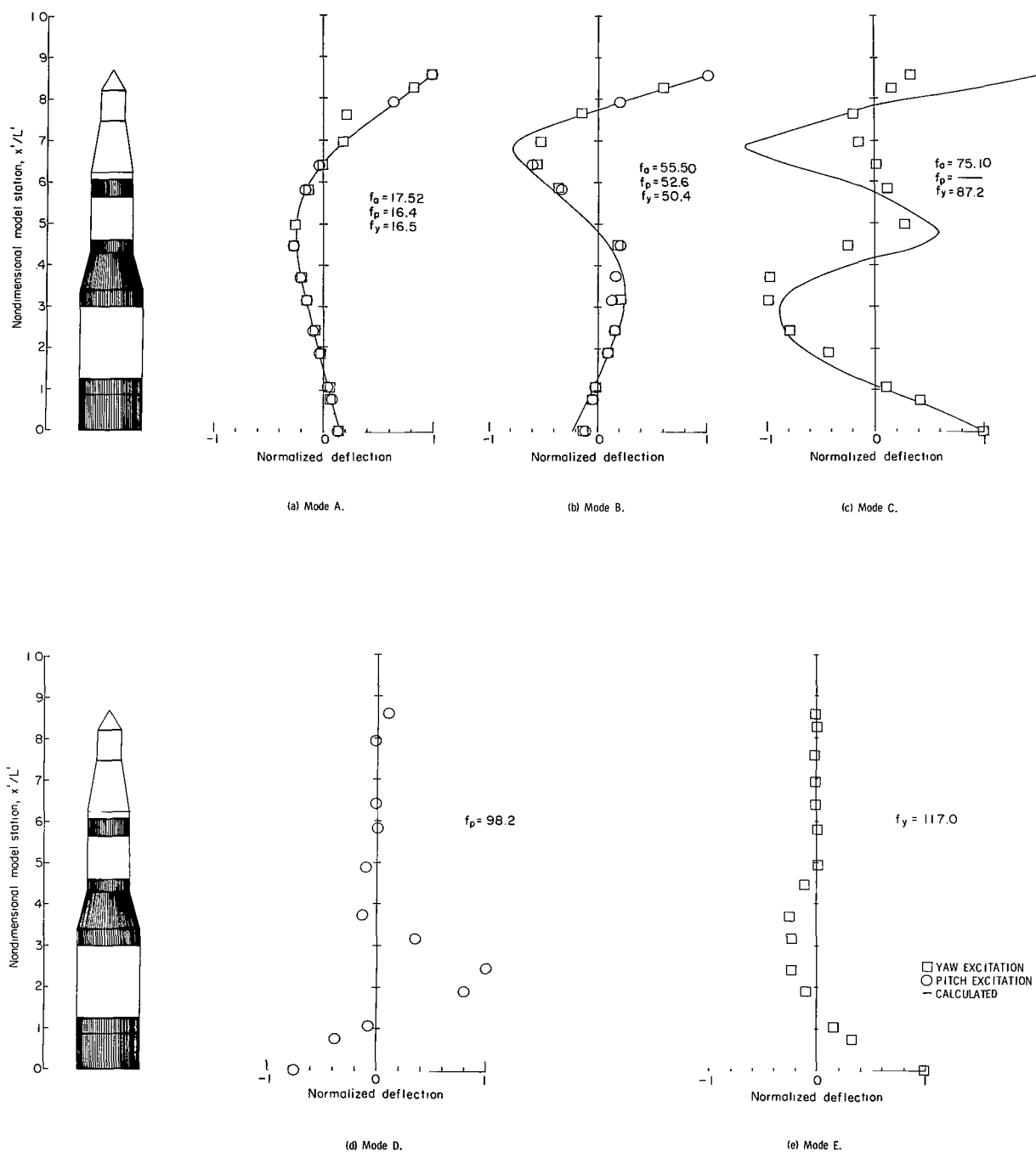


Figure 17.- Measured and calculated deflection shapes for configuration II with 100 percent propellant.

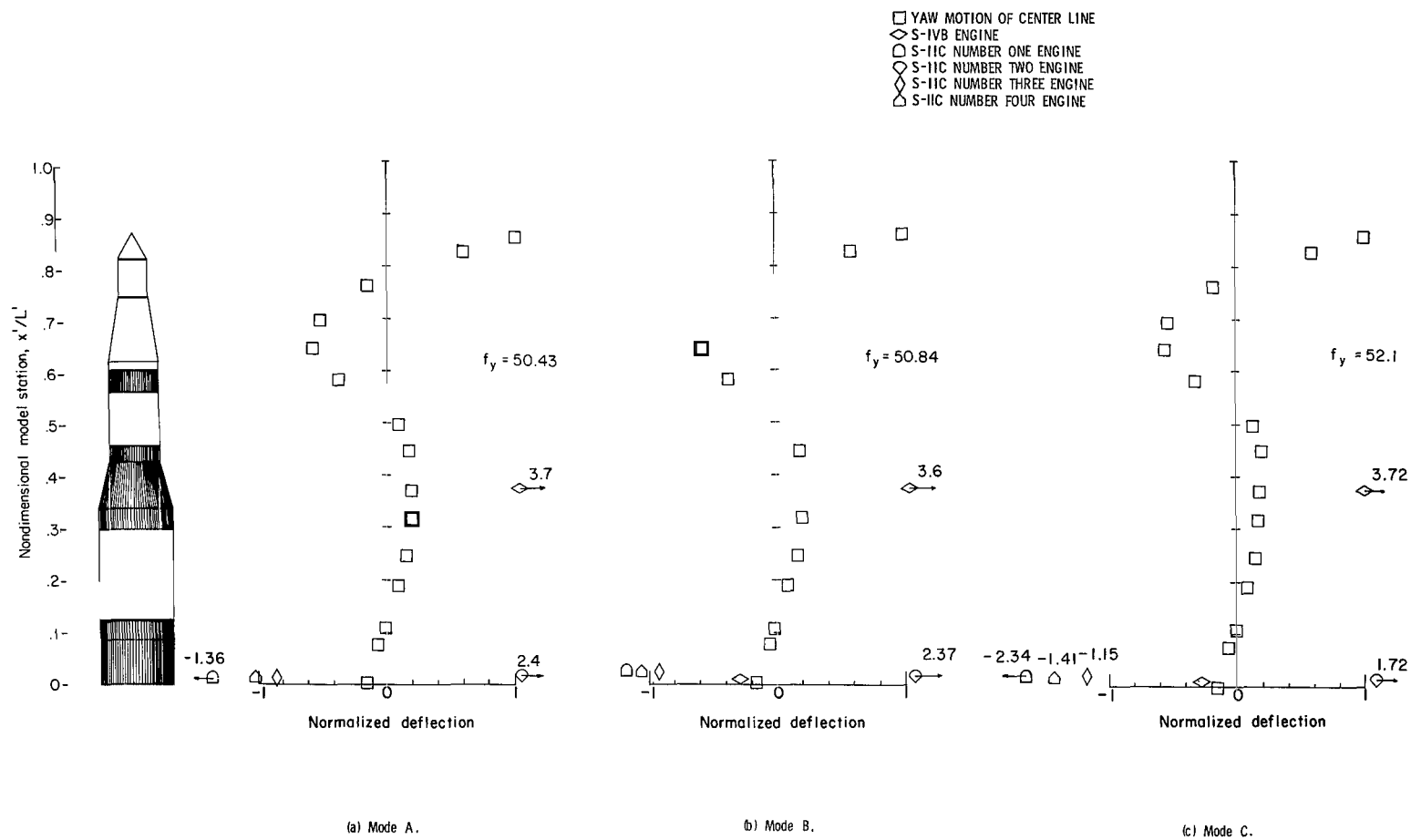
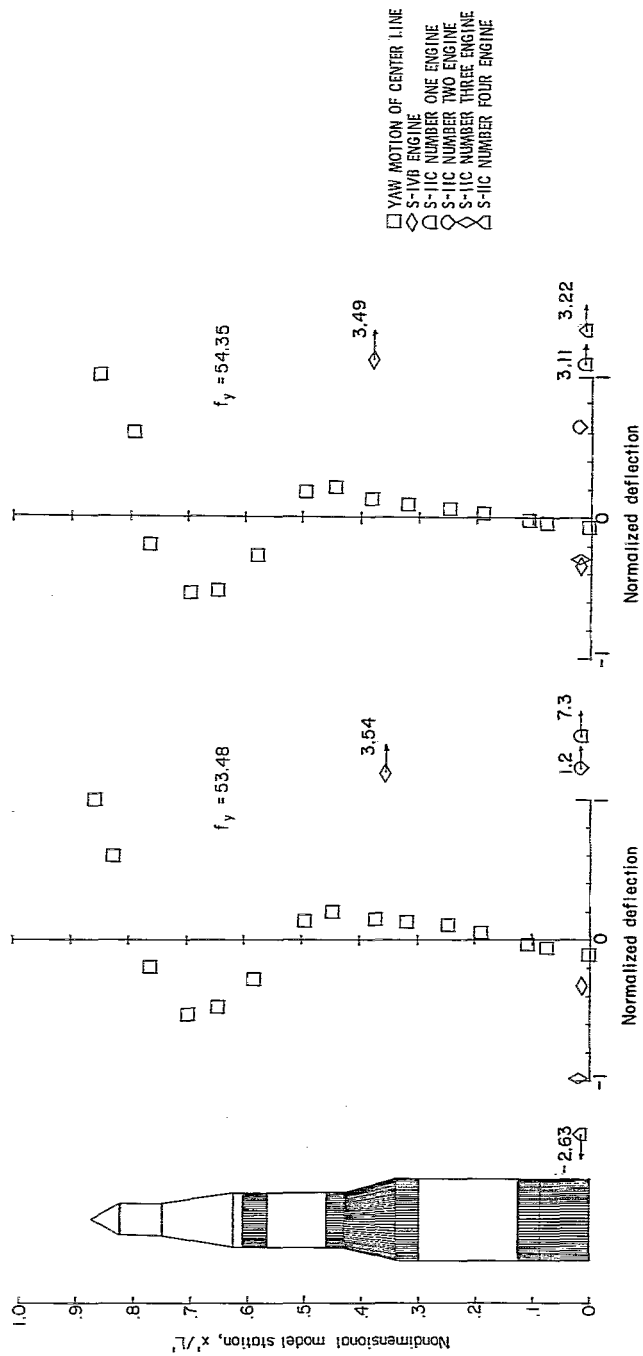


Figure 18.- Measured engine resonance deflection shapes for configuration II with 100 percent propellant.



(e) Mode E.

(d) Mode D.

Figure 18.- Concluded.

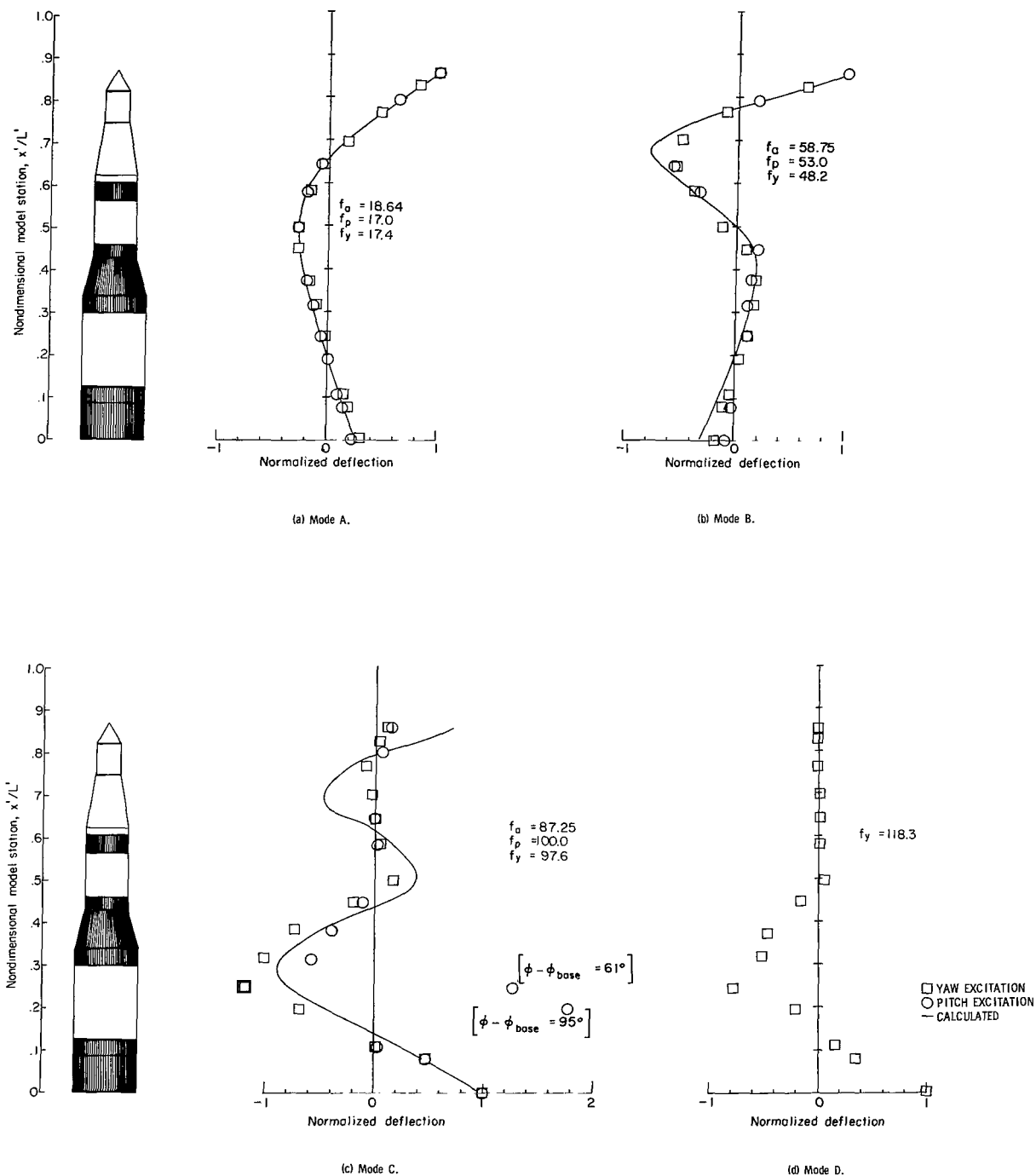


Figure 19.- Measured and calculated deflection shapes for configuration 11 with 50 percent propellant.

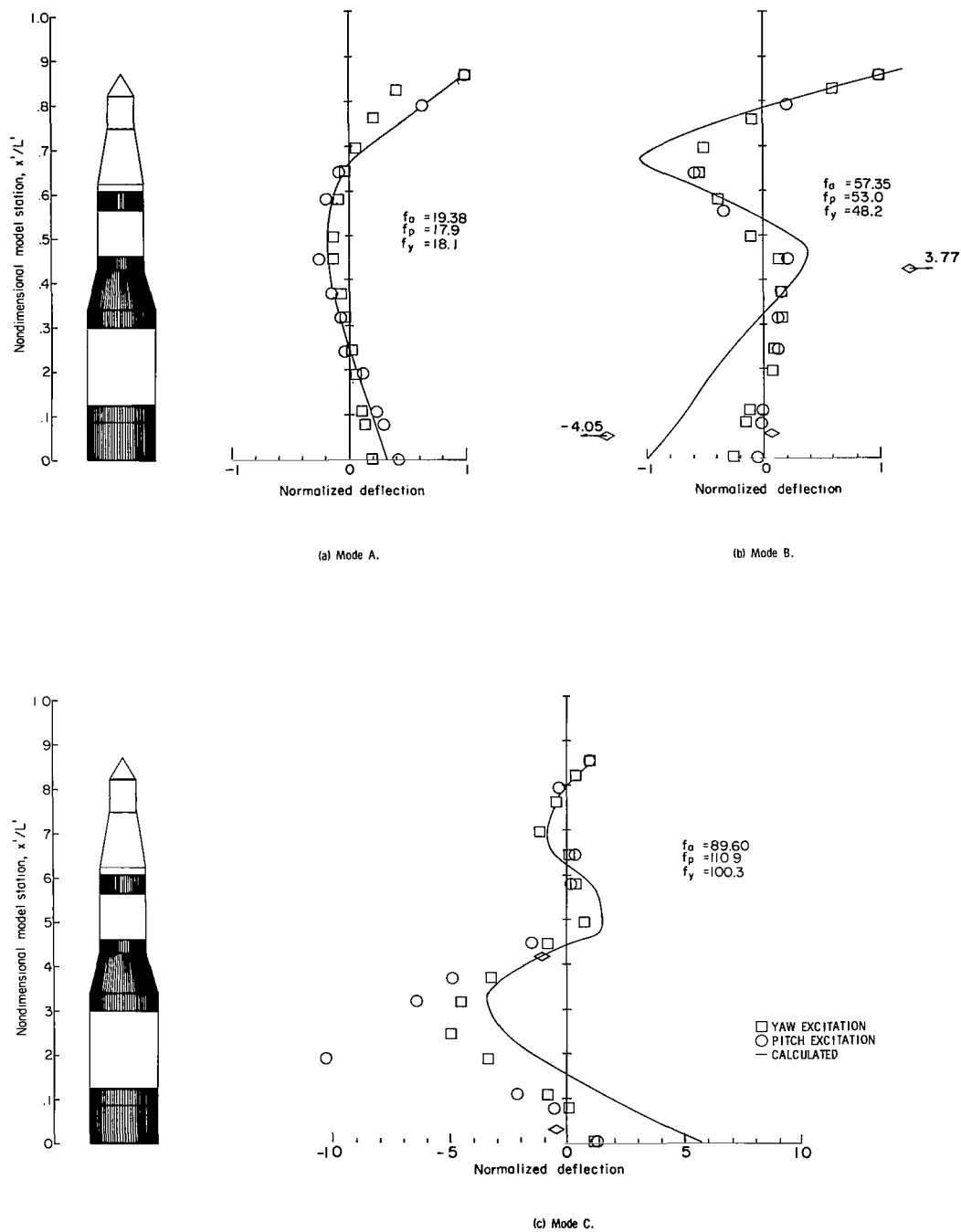


Figure 20.- Measured and calculated deflection shapes for configuration 11 with 0 percent propellant.

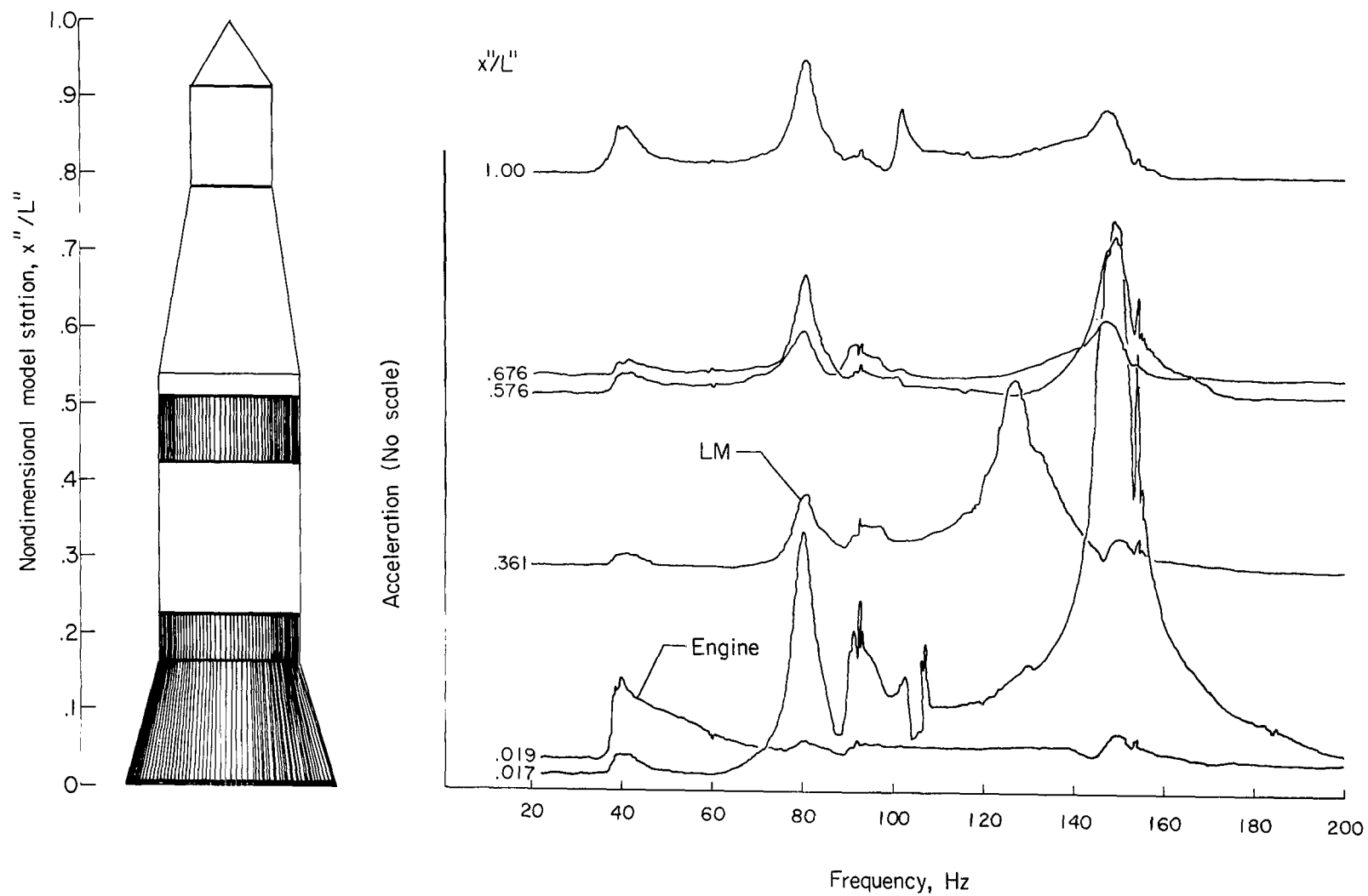


Figure 21.- Accelerations of configuration III with 100 percent propellant as a function of frequency for excitation in the yaw direction.

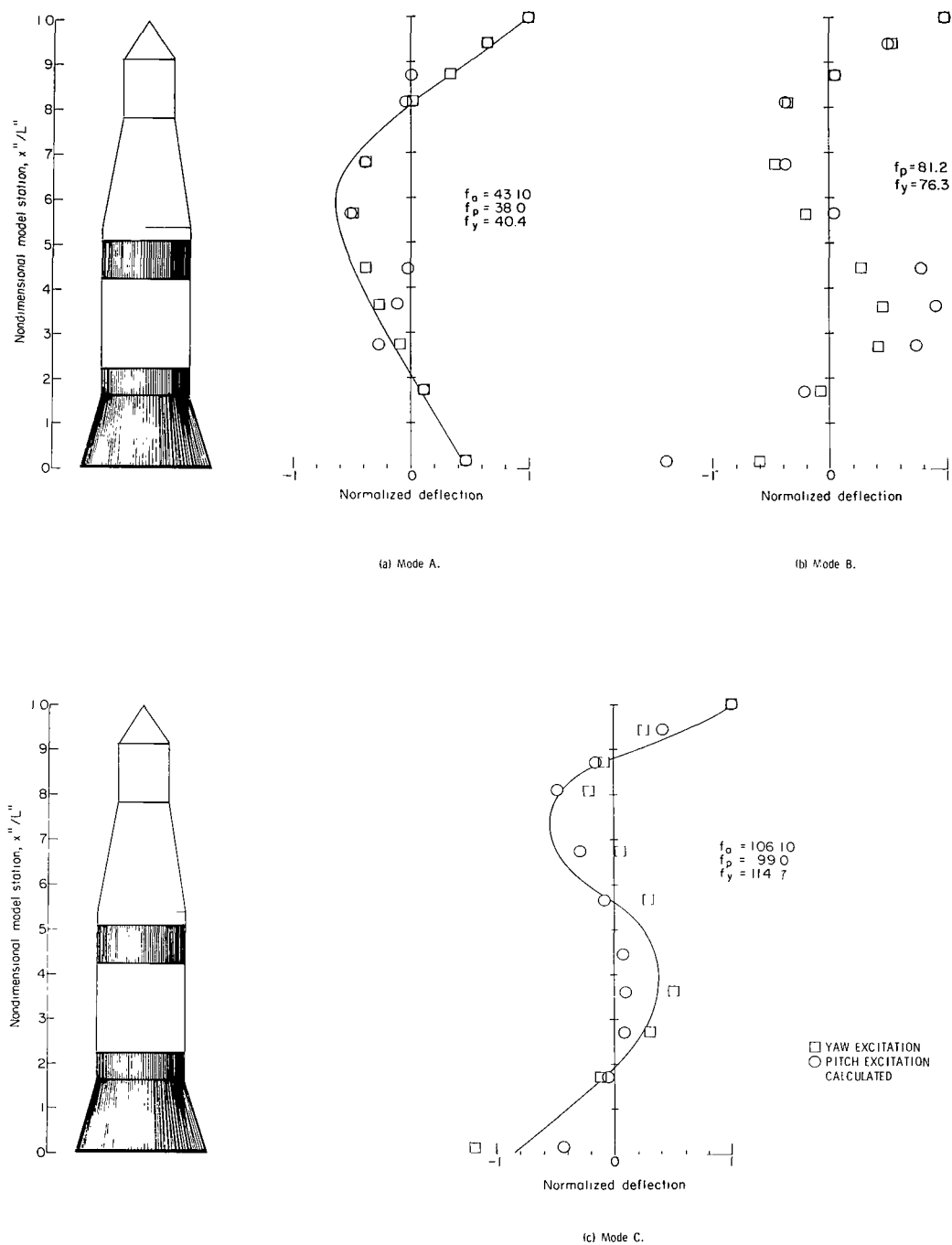
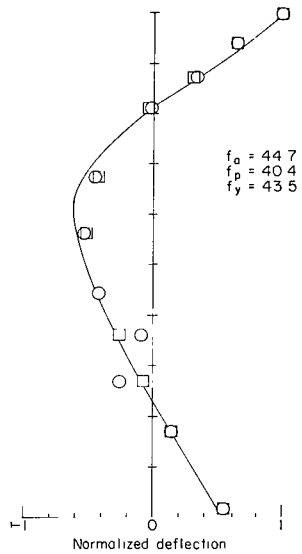
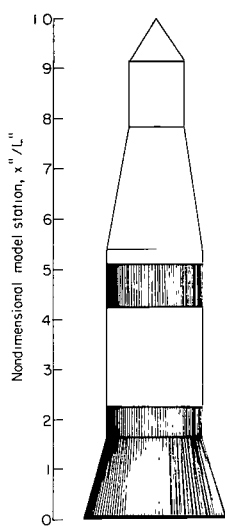
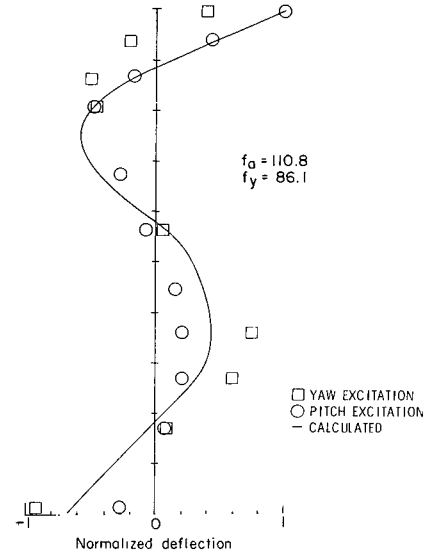


Figure 22.- Measured and calculated deflection shapes for configuration III with 100 percent propellant.



(a) Mode A.



(b) Mode B.

Figure 23.- Measured and calculated deflection shapes for configuration III with 50 percent propellant.

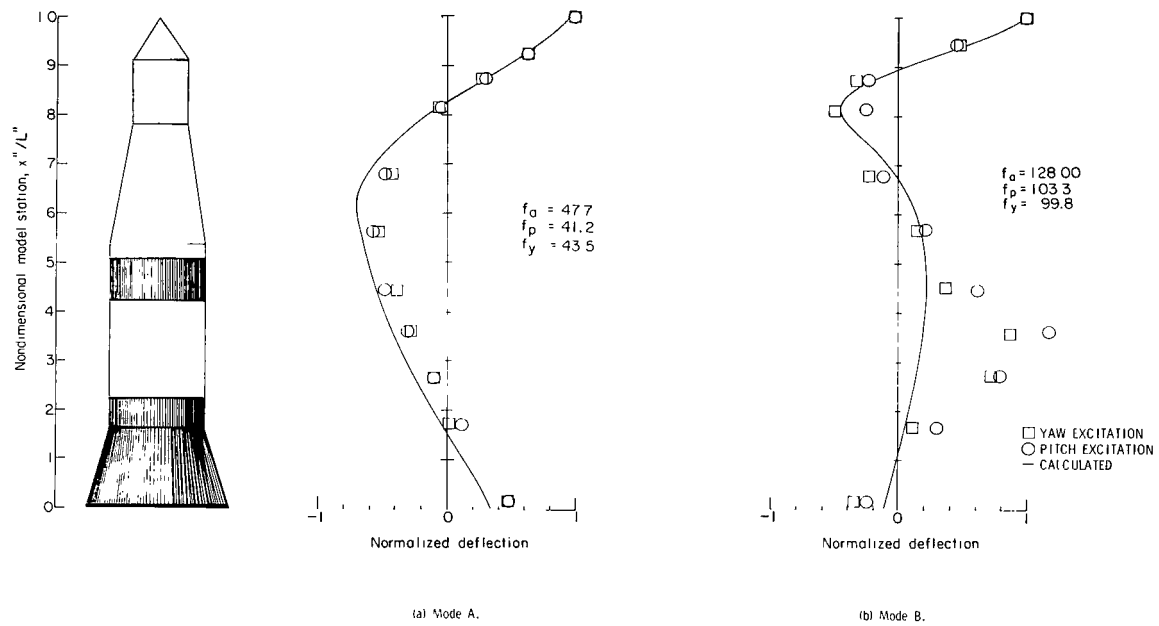


Figure 24.- Measured and calculated deflection shapes for configuration III with 0 percent propellant.

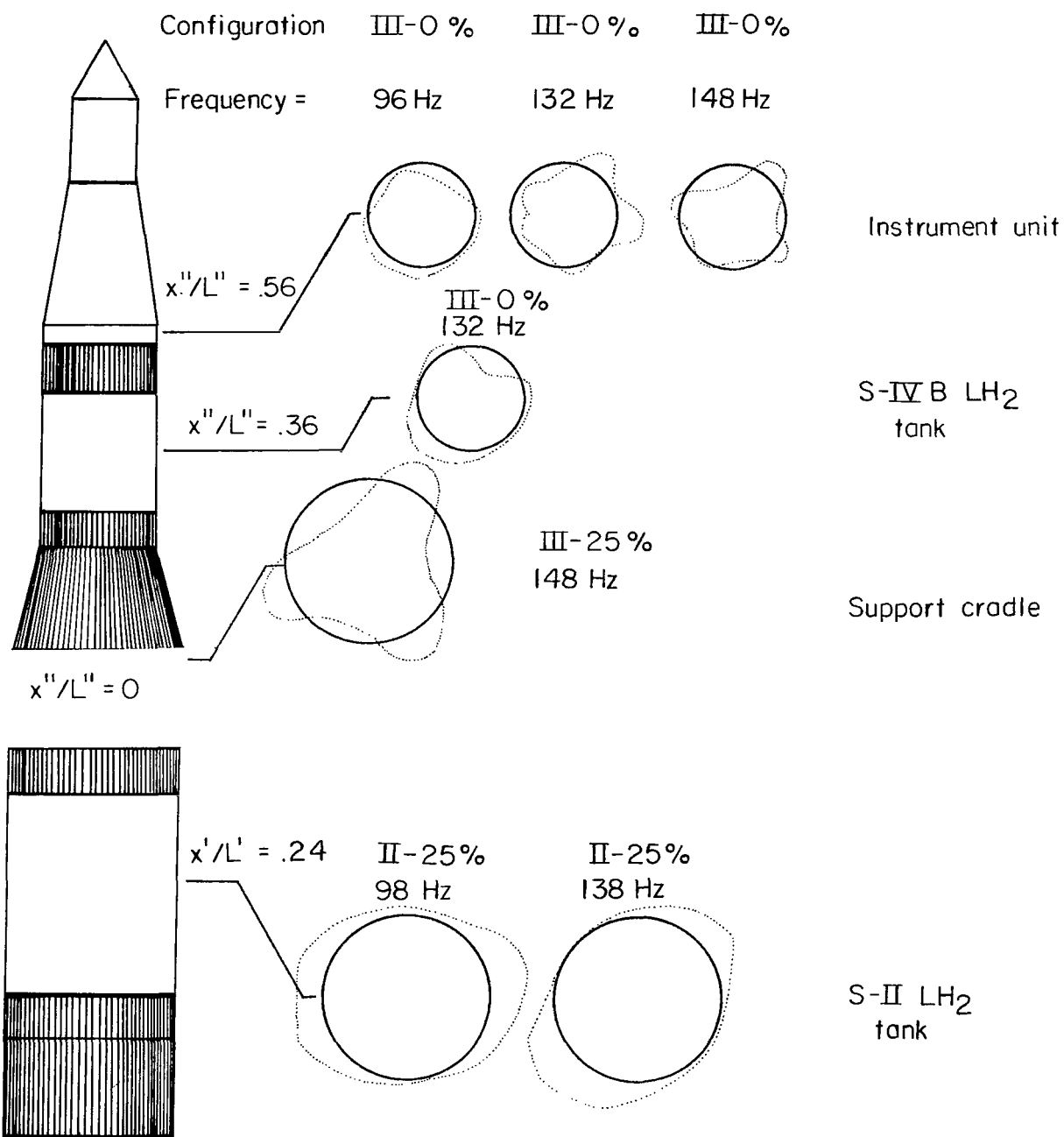


Figure 25.- Typical measured shell deformations.

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